

Lesson #4

Finding the Slope and Y-Intercept of a Line Given 2 Points

What are Slope and Y-Intercept?

- The **slope** of a line is a numerical measure of how steep the line is and in what direction it rises
- The **y-intercept** of a line is the point on the coordinate plane (or the x-y grid) where the line crosses the y-axis
- The y-intercept is often expressed as a point (0, #)

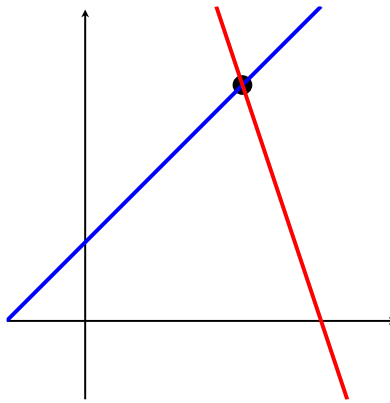


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Slope-Intercept Form of a Line

A **Line in Slope-Intercept Form** is expressed in the form $y = mx + b$, where m and b are any real numbers and can be either positive or negative

The advantage of this form is that we can immediately tell the slope (**m**) of a line and its y-intercept (**b**)

To find the equation of a line given 2 points, we begin by finding the SLOPE and then proceed to find the Y-INTERCEPT

Finding the Slope of A Line Given 2 Points

When you are given 2 points (x_1, y_1) and (x_2, y_2) the slope formula is:

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example: Find the slope of the line containing the points $(-1, -5)$ and $(3, 9)$

Solution: We have $x_1 = -1$, $y_1 = -5$, $x_2 = 3$ and $y_2 = 9$

Substituting into the equation and simplifying gives:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - (-5)}{3 - (-1)} = \frac{14}{4} = \frac{7}{2}$$

Now We Find the Y-Intercept...

Once we have the slope (m), we need to find the y-intercept (b). We do this by using the slope, one point (x, y) and the equation $y = mx + b$

Example: Find the y-intercept for the line containing the points $(-1, -5)$ and $(3, 9)$

Solution: First we need to find the slope (m), which we found above:

$$m = \frac{7}{2}$$

We can choose **either** point. Let's use $(3, 9)$

So we have $m = \frac{7}{2}$. By choosing the point $(3, 9)$, $x = 3$ and $y = 9$

Substitute these values into $y = mx + b$ and solve for **b**:

$$y = mx + b$$

$$9 = \frac{7}{2}(3) + b$$

$$9 = \frac{21}{2} + b$$

$$\frac{18}{2} = \frac{21}{2} + b$$

$$-\frac{3}{2} = b$$

Putting It All Together...

- Now that we have our slope m and our y-intercept b , we insert them into the slope-intercept form of the line: $y = mx + b$
- For our example, $m = \frac{7}{2}$ and $b = -\frac{3}{2}$
- Our final equation, then, is $y = \frac{7}{2}x - \frac{3}{2}$

Note that sometimes a problem will ask for the equation. Other times, you will be asked for the slope, y-intercept or both. Be sure to read the question and answer choices carefully!

Now For Some Practice...

Sample Question: What is the equation of the line through the points (3, -1) and (6, 8) in slope intercept form?

Solution: We begin by using the slope formula with

$$x_1 = 3, y_1 = -1, x_2 = 6 \text{ and } y_2 = 8:$$

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - (-1)}{6 - 3} = \frac{9}{3} = 3$$

Now, we need to find the y-intercept (b)

We can choose either point. Let's use (3, -1). So $x = 3$ and $y = -1$

With $m = 3$, $x = 3$ and $y = -1$, we substitute into $y = mx + b$:

$$y = mx + b$$

$$-1 = 3(3) + b$$

$$-1 = 9 + b$$

$$\mathbf{-10 = b}$$

With $m = 3$ and $b = -10$ we substitute into the final equation:

$$y = mx + b$$

$$\mathbf{y = 3x - 10}$$