

Lesson #7

Finding the Greatest Common Factor (GCF)

What Is A Greatest Common Factor (GCF)?

The GCF of two numbers is the largest number that divides evenly into those two numbers. For example, the GCF for 50 and 20 is 10.

It is also possible to find the GCF of variable expressions:

For example, the GCF for x^3y and x^2y^3 is x^2y

Finally, we can find the GCF for expressions that have both numbers and variables.

For example, the GCF for $27a^3b^2$ and $18ab^3$ is $9ab^2$

What is a Prime Factorization?

The prime factorization of a number is found by expressing a number as a series of prime numbers multiplied together.

A prime number is any number which is only divisible by itself and 1. Basically, any number which can be divided by another integer (that isn't 1) is NOT PRIME. **The first prime number is 2 (not 1!).**

Here are the first 10 prime numbers: **2, 3, 5, 7, 11, 13, 17, 19, 23, 29**

How to Find the GCF for 2 Numbers

There are multiple ways to find the GCF of 2 numbers. In this lesson, we will begin by writing out the prime factorizations of the 2 numbers.

- Note: There are multiple ways to write the factorization for a number and as long as you reduce completely, you will always end up with the same prime factorization.

For example, two ways to find the SAME prime factorization of 54 are:

$$54 = 2 * 27 = 2 * 3^3 = 2 * 3 * 3 * 3$$

$$54 = 6 * 9 = 2 * 3 * 3^2 = 2 * 3 * 3 * 3$$

Sample Problem: Find the GCF for 54 and 120

Begin by expanding each of the numbers:

$$54 = 2 * 27 = 2 * 3^3 = 2 * 3 * 3 * 3$$

$$120 = 8 * 15 = 2^3 * 3 * 5 = 2 * 2 * 2 * 3 * 5$$

Now, to find the GCF, highlight any factors that exist in both prime factorizations (multiples are ok if they appear in both numbers):

$$54 = 2 * 3 * 3 * 3$$

$$120 = 2 * 2 * 2 * 3 * 5$$

The last step is to MULTIPLY the shared factors:

$$\text{The GCF for 54 and 120 is } 2 * 3 = 6$$

What If There Are Variables Involved? No Problem!!!

If an expression contains variables, simply write out each power of the variable(s) as its own factor. For example, we would write the factorization of x^4 as $x * x * x * x$

If numbers and variables are combined together, it is still not difficult. The prime factorization of $42a^3b^2$ is:

$$42 * a * a * a * b * b$$

$$= 6 * 7 * a * a * a * b * b$$

$$= 2 * 3 * 7 * a * a * a * b * b$$

How to Find the GCF For 3 Numbers

Sometimes you will be asked to find the Greatest Common Factor for 3 Numbers. There are 2 ways to accomplish this:

- A) Write out the prime factorizations for all of the numbers and then find shared factors for ALL 3 numbers
- B) Find the GCF for the first 2 numbers. Then take that GCF and the 3rd number, and find the GCF for those 2 numbers

The next 2 examples show the same problem solved in each of these different ways. You can choose which one you prefer!

Finding the GCF for 3 Numbers – Method A

Sample Problem: Find the GCF for 78, 84 and 135

Solution: Begin by expanding each of the numbers:

$$78 = 2 * 39 = 2 * 3 * 13$$

$$84 = 2 * 42 = 2 * 6 * 7 = 2 * 2 * 3 * 7$$

$$135 = 27 * 5 = 3^3 * 5 = 3 * 3 * 3 * 5$$

Now, to find the GCF, highlight any factors that exist in all 3 prime factorizations (multiples are ok if they appear in all 3 numbers)

$$78 = 2 * 3 * 13$$

$$84 = 2 * 2 * 3 * 7$$

$$135 = 3 * 3 * 3 * 5$$

3 is the only shared factor for all three numbers

Therefore, the GCF for 78, 84 and 135 is 3

Finding the GCF for 3 Numbers – Method B

Sample Problem: Find the GCF for 78, 84 and 135

Solution: Step One: Find the GCF for the first 2 numbers by highlighting any factors that exist in both prime factorizations:

$$78 = 2 * 3 * 13$$

$$84 = 2 * 2 * 3 * 7$$

The GCF for the first 2 numbers is $2 * 3 = 6$

Step Two: Now find the GCF for 6 and 135:

$$6 = 2 * 3$$

$$135 = 3 * 3 * 3 * 5$$

The GCF for these 2 numbers is 3

Therefore, the GCF for 78, 84 and 135 is 3

Now For Some Practice...

Sample Question: What is the greatest common factor of 64 and 120?

Solution: Begin by finding the prime factorizations for each number:

$$64 = 16 * 4 = 4 * 4 * 4 = 2 * 2 * 2 * 2 * 2 * 2$$

$$120 = 8 * 15 = 2^3 * 3 * 5 = 2 * 2 * 2 * 3 * 5$$

Highlight the shared factors:

$$64 = 2 * 2 * 2 * 2 * 2 * 2$$

$$120 = 2 * 2 * 2 * 3 * 5$$

Now multiply the shared factors: $2 * 2 * 2 = 8$

The GCF for 64 and 120 is 8

Sample Question: What is the greatest common factor for $32x^2$ and $52x^5$?

Solution: Begin by finding the prime factorizations for each expression:

$$32x^2 = 8 * 4 * x^2 = 2 * 4 * 4 * x * x = 2 * 2 * 2 * 2 * 2 * x * x$$

$$52x^5 = 2 * 26 * x^5 = 2 * 2 * 13 * x * x * x * x * x$$

Highlight the shared factors:

$$32x^2 = 2 * 2 * 2 * 2 * 2 * x * x$$

$$52x^5 = 2 * 2 * 13 * x * x * x * x * x$$

Now multiply the shared factors: $2 * 2 * x * x = 4x^2$

The GCF for the two expressions is $4x^2$