

Lesson #3

Finding the Other Endpoint of a Segment When You Are Given the Midpoint and One Endpoint

What Is A Midpoint and How Do We Find One?

A Midpoint is the point which divides a line segment into two smaller, equal segments

Or.... the “halfway point” of a line segment

Given 2 points, (x_1, y_1) and (x_2, y_2) , the **Midpoint Formula** is:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Using The Midpoint Formula

Sample Question: What is the midpoint of the segment with endpoints $(-3, 6)$ and $(2, -4)$?

Solution: Use the midpoint formula with $x_1 = -3$, $y_1 = 6$, $x_2 = 2$, and $y_2 = -4$

$$\begin{aligned} \text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \\ &= \left(\frac{-3 + 2}{2}, \frac{6 + (-4)}{2} \right) \\ &= \left(\frac{-1}{2}, \frac{2}{2} \right) \\ &= \left(-\frac{1}{2}, 1 \right) \end{aligned}$$

But What If We Know the *Midpoint* and Need to Find the **OTHER** Endpoint?

- Don't panic!!! We just need to use the midpoint formula in a slightly different way
 - This time, we know the coordinates of the MIDPOINT and ONE of the endpoints
 - We work backwards with the formulas and it all works out just fine
 - The key is knowing *WHAT YOU HAVE* and *WHAT YOU NEED*!
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Using The Midpoint Formula Creatively

Sample Question: C is the midpoint of segment $\overline{AB}$. If the coordinates of point A are (-3, 4) and point C are (7, 12), what are the coordinates of point B?

Solution: This time we are given the midpoint and one endpoint. Let's assign variables to the midpoint (x_m, y_m) . In this scenario, $x_m=7$ and $y_m=12$. For our given endpoint A, $x_1=-3$ and $y_1=4$.

Again, we have $x_m=7$, $y_m=12$, $x_1=-3$ and $y_1=4$

We work with the x and y coordinates separately using the midpoint formula:

$$(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\begin{aligned} \bullet \quad x_m &= \frac{x_1 + x_2}{2} \\ \bullet \quad 7 &= \frac{-3 + x_2}{2} \\ \bullet \quad 14 &= -3 + x_2 \\ \bullet \quad 17 &= x_2 \end{aligned}$$

$$\begin{aligned} \bullet \quad y_m &= \frac{y_1 + y_2}{2} \\ \bullet \quad 12 &= \frac{4 + y_2}{2} \\ \bullet \quad 24 &= 4 + y_2 \\ \bullet \quad 20 &= y_2 \end{aligned}$$

So, the coordinates of Endpoint B = (x_2, y_2) are (17, 20)

Sample Question: The midpoint of a segment is $(-2, 4)$. If one endpoint is $(6, -3)$, find the coordinates of the other endpoint.

Solution: This time we are given the midpoint and one endpoint. Let's assign variables to the midpoint (x_m, y_m) . In this scenario, $x_m = -2$ and $y_m = 4$. For our given endpoint, $x_1 = 6$ and $y_1 = -3$.

Again, we have $x_m = -2$, $y_m = 4$, $x_1 = 6$ and $y_1 = -3$

We work with the x and y coordinates separately using the midpoint formula:

$$(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

- $x_m = \frac{x_1 + x_2}{2}$
- $-2 = \frac{6 + x_2}{2}$
- $-4 = 6 + x_2$
- $-10 = x_2$

- $y_m = \frac{y_1 + y_2}{2}$
- $4 = \frac{-3 + y_2}{2}$
- $8 = -3 + y_2$
- $11 = y_2$

So, the coordinates of the Endpoint = $(x_2, y_2) = (-10, 11)$