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WORKBOOK

A Unique Set of Lessons Designed to Prepare You For the ACT Math Section!

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ABOUT THIS WORKBOOK…

These lessons focus on the fundamentals and commonly asked types of ACT Math questions. After explaining the concepts, we will show you how to harness the power of your TI-83 / TI-84 calculator to quickly and accurately solve problems and check your answers. This should give you more time to focus on the more complex ACT math problems that often go unanswered.

- This workbook was designed as a companion to our unique product, Easy-To-Use Graphing Calculator Programs: 15 **ACT-Compliant** programs which assist you with solving some of the most commonly tested types of questions.
- These programs are coded for you and delivered so that they can be easily loaded onto your TI-83/84 graphing calculator.
- For information about obtaining these programs, please visit our website at http://calculatesuccess.org

PRE-TEST

How To Take This Pre-Test

- The purpose of this pre-test is to assess your skills and knowledge at this time
- Allow yourself about 45 minutes to take the pre-test
- You may use a calculator for any/all of the problems
- This assessment shows you where you are at right now
- Do NOT be discouraged if you struggle with any or all of these questions – our course is designed to help you master these types of problems!

Before You Begin…

- Each question has exactly one correct answer.
- If you are unsure of the correct answer, take your best guess.
- You are not penalized for guessing on the ACT.
- Allow yourself approximately 45 minutes to complete this pre-test.

1. Find the solutions to the quadratic equation: $x^2 - 7x + 4 = 0$

   A. -6.372 and -.628

   B. -6.372 and .628

   C. -.628 and 6.372

   D. .628 and 6.372

   E. No Real Solutions

2. Evaluate the expression for x = -3: $-x^3 - 2x^2 + 5|x|$

   A. -60

   B. -24

   C. 6

   D. 24

   E. 30

3. One interior angle of a regular polygon measures 144 degrees. What is the name of the polygon?

   A. Pentagon

   B. Hexagon

   C. Octagon

   D. Decagon

   E. Dodecagon

4. What is the slope of the line containing the points (-6, 4) and (3, -7)?

   A. $-\frac{11}{9}$

   B. $-\frac{9}{11}$

   C. $\frac{9}{11}$

   D. 1

   E. $\frac{11}{9}$

5. 42 is 40% of what number?

   A. 0.0095

   B. 16.8

   C. 105

   D. 168

   E. 1680

6. Factor the polynomial expression: $x^2 + 4x + 5$

   A. The polynomial cannot be factored

   B. $(x - 1)(x - 4)$

   C. $(x - 1)(x + 5)$

   D. $(x + 1)(x + 4)$

   E. $(x + 1)(x + 5)$

7. Rebecca used a 30% off coupon to purchase a television that was originally priced at $699. How much did she pay?

   A. $209.70

   B. $489.30

   C. $719.97

   D. $908.70

   E. $2330.00

8. What is the distance between the points (2, -4) and (-7, 9), rounded to the nearest hundredth?

   A. 7.07

   B. 9.38

   C. 10.30

   D. 13.93

   E. 15.81

9. The midpoint of a segment is (-2, 4). If one endpoint is (6, -3), find the coordinates of the other endpoint.

   A. $(-10,11)$

   B. $(-\frac{1}{2},2)$

   C. $(2,\frac{1}{2})$

   D. $(14,-10)$

   E. $(14,-7)$

10. The hypotenuse of a right triangle is 12 units. One leg is 9 units. Find the other leg.

   A. 3

   B. 7.94

   C. 9.74

   D. 15

   E. 21

11. What is the greatest common factor for the two terms: $32x^2\ and\ 52x^5$

   A. 4

   B. $4x$

   C. $4x^2$

   D. $16x$

   E. $16x^2$

12. Bob has received test grades of 84, 92, 89 and 74. What will he need on the fifth test to achieve a mean of 86?

   A. 79

   B. 84

   C. 86

   D. 88

   E. 91

13. What is the y-intercept of the line -4x + 6y = 30 ?

A. -7.5

B. -5

C. -1.5

D. 5

E. 7.5

14. What is the measure of one exterior angle in a regular 10-sided polygon?

A. 36°

B. 54°

C. 90°

D. 144°

E. 180°

15. R is the midpoint of segment ST. If the coordinates of point S are (-2, 7) and point T are (0, -5), what are the coordinates of point R?

A. (-1, 1)

B. (-1, 6)

C. (1, -6)

D. (1, -1)

E. (1, 1)

16. Evaluate the mathematical expression for a = -1 and b = 3: $ab^2 - 2b^a + 3a$

A. $-\frac{38}{3}$

B. 0

C. $\frac{16}{3}$

D. 6

E. 14

17. What is the equation of the line through the points (3, -1) and (6, 8) in slope intercept form?

A. $y = -3x + 10$

B. $y = \frac{1}{3}x - 10$

C. $y = \frac{1}{3}x + 5$

D. $y = \frac{7}{9}x - 2$

E. $y = 3x - 10$

18. What is the complete factorization for the polynomial: $-3x^2 + 33x - 30$

A. $-3(x - 10)(x - 1)$

B. $-3(x + 10)(x + 1)$

C. $(-3x + 3)(x - 10)$

D. $(3x - 3)(x + 10)$

E. $3(-x + 1)(x - 10)$

19. What is the sum of the measures of the exterior angles for an 11-sided polygon?

A. 360°

B. 720°

C. 1080°

D. 1440°

E. 1620°

20. What is the solution to the system of equations: $\begin{cases} 3x - 2y = 12 \\ -7x + 4y = -26 \end{cases}$

A. (-3, -2)

B. (-3, 2)

C. (-2, 3)

D. (2, -3)

E. (3, -2)

21. What is the greatest common factor for 78, 84 and 135?

A. 1

B. 3

C. 6

D. 9

E. 12

22. Find the solutions to the quadratic equation: $6x^2 + 28x = 0$

A. $-\frac{1}{3}\ and\ 4$

B. $-\frac{14}{3}\ and\ 0$

C. $No\ real\ solutions$

D. $0\ and\ \frac{14}{3}$

E. $\frac{1}{3}\ and\ 4$

23. For his first 6 golf games, Nick has a total combined score of 462. What score would he need on his next golf game to have an average score of 76?

A. 64

B. 67

C. 70

D. 73

E. 76

24. Find the distance between the points (-1, 4) and (3, 7).

A. 5

B. 7

C. 10

D. 13

E. 25

25. What number is 60% of 75?

A. 30

B. 45

C. 125

D. 240

E. 450

26. A line contains the points (0, -2) and (3, 7). What is the slope of a line perpendicular to this line?

A. -3

B. $-\frac{1}{3}$

C. $\frac{1}{3}$

D. $\frac{2}{3}$

E. 3

27. Find the solutions to the quadratic equation: $9x^2 - 20 = 0$

A. -1.491 and 1.262

B. -1.491 and 1.491

C. -1.262 and 1.262

D. -1.262 and 1.491

E. No real solutions

28. A customer wants to leave an 18% tip on a dinner bill in the amount of $75. What is the total cost of his meal, including the tip?

A. $61.50

B. $76.35

C. $88.50

D. $93.00

E. $135.00

29. The sum of the interior angles of a polygon is 1980 degrees. How many sides does the polygon have?

A. 10

B. 11

C. 12

D. 13

E. 14

30. Evaluate the mathematical expression for r = 3 and s = -6: $\frac{6rs^{-1}}{3r^2s}$

A. -432

B. $-\frac{1}{27}$

C. $\frac{1}{54}$

D. $\frac{2}{3}$

E. 54

31. Factor the polynomial completely: $5x^2 - 30x - 20$

A. $-5(x^2 + 6x + 4)$

B. $5(x^2 - 6x - 4)$

C. $5(x^2 + 6x - 4)$

D. $5(x - 2)(x + 2)$

E. $(5x - 1)(x + 4)$

32. What is the greatest common factor of 64 and 120?

A. 2

B. 4

C. 8

D. 16

E. 32

33. How many real solutions does the quadratic equation have: $x^2 + 2x + 5 = 0$

A. 0

B. 1

C. 2

D. 3

E. 4

34. Find the slope of the line containing the points (-1, -5) and (3, 9).

A. -1

B. $-\frac{2}{7}$

C. $\frac{2}{7}$

D. 1

E. $\frac{7}{2}$

35. Sadie has an average of 84% after taking 8 chemistry tests. What grade would she need on her next test to bump her average up to 85%?

A. 84%

B. 85%

C. 88%

D. 93%

E. 96%

36. What is the midpoint of the segment with endpoints (-3, 6) and (2, -4)?

A. $\left(-\frac{5}{2}, 5\right)$

B. $\left(-\frac{1}{2}, 1\right)$

C. $\left(\frac{1}{2}, 1\right)$

D. $\left(\frac{3}{2}, 1\right)$

E. $\left(\frac{5}{2}, -5\right)$

37. The legs of a right triangle have lengths 5 cm and 12 cm. What is the length of the hypotenuse?

A. 7 cm

B. 12 cm

C. 13 cm

D. 15 cm

E. 17 cm

38. 75 is what percent of 300?

A. 2.5%

B. 25%

C. 30%

D. 35%

E. 250%

39. What is the slope of the line 6x - 3y = 18 ?

A. -2

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. 2

E. 3

40. What is the correct factorization for the polynomial: $x^2 - 11x - 60$

A. $(x - 15)(x + 4)$

B. $(x - 12)(x + 5)$

C. $(x - 10)(x - 6)$

D. $(x - 10)(x + 6)$

E. $(x - 4)(x + 15)$

41. What is the measure of one interior angle in a regular octagon?

A. 47°

B. 67.5°

C. 90°

D. 120°

E. 135°

42. C is the midpoint of segment AB. If the coordinates of point A are (-3, 4) and point C are (7, 12), what are the coordinates of point B?

A. $(-13, -4)$

B. $(2, 8)$

C. $(5, 4)$

D. $(16, 18)$

E. $(17, 20)$

43. What is the sum of the solutions to the quadratic equation: $8x^2 + 2x - 15 = 0$

A. The equation has no real solutions

B. -4

C. $-\frac{1}{4}$

D. $\frac{1}{4}$

E. 4

44. Evaluate the mathematical expression: $-3^2 - 2(5-8) * 9/3$

A. -27

B. -24

C. -9

D. 9

E. 27

45. A regular polygon has exterior angles which each measure 45 degrees. How many sides does the polygon have?

A. 6

B. 7

C. 8

D. 9

E. 10

46. For the solution to the system of equations, what is the value of x – y: $\begin{cases} -5x - 3y = 9 \\ 2x + 7y = 8 \end{cases}$

A. -6

B. -5

C. -1

D. 1

E. 5

PRE-TEST SOLUTIONS

1.	D	13.	D	25.	B	37.	C
2.	D	14.	A	26.	B	38.	B
3.	D	15.	A	27.	B	39.	D
4.	A	16.	A	28.	C	40.	A
5.	C	17.	E	29.	D	41.	E
6.	A	18.	A	30.	C	42.	E
7.	B	19.	A	31.	B	43.	C
8.	E	20.	D	32.	C	44.	D
9.	A	21.	B	33.	A	45.	C
10.	B	22.	B	34.	E	46.	B
11.	C	23.	C	35.	D		
12.	E	24.	A	36.	B		

Tutorial #1

Finding the Score Needed to Obtain a Particular Mean (Average)

The general formula for finding a mean is:

$$Mean = \frac{Sum\ of\ Scores}{Number\ of\ Scores}$$

However, we are often asked more complicated questions, like this:

- "Bob has received test grades of 84, 92, 89 and 74. What will he need on the fifth test to achieve a mean of 86?"
- "For his first 6 golf games, Nick has a total combined score of 462. What score would he need on his next golf game to have an average score of 76?"
- "Sadie has an average of 84% after taking 8 chemistry tests. What grade would she need on her next test to bump her average up to 85%?"

The key to solving these more involved problems is to use a slightly more complicated formula. You will need to know 3 things:

A) What is our desired mean/average?

B) What is the sum of all the scores we have?

C) How many scores/test grades will we have?

$$\boldsymbol{Desired\ Mean = \frac{Current\ Sum\ of\ Scores + x}{Total\ Number\ of\ Scores}}$$

Where x = the final score needed to achieve the desired mean

Using the Formula

Sample Question: Bob has received test grades of 84, 92, 89 and 74. What will he need on the fifth test to achieve a mean of 86?

Solution: We gather the required information. Our desired mean is **86**. The sum of the scores we have is 84+92+89+74 = **339**. We have 4 scores and want the mean for a total of **5** tests.

We now substitute this info into our formula and solve for x:

$$Desired\ Mean = \frac{Current\ Sum\ of\ Scores + x}{Total\ Number\ of\ Scores}$$

$$86 = \frac{339 + x}{5}$$

$$430 = 339 + x$$

$$91 = x$$

Bob needs a score of 91 on his fifth test to achieve a mean of 86

Sample Question: For his first 6 golf games, Nick has a total combined score of 462. What score would he need on his next golf game to have an average score of 76?

Solution: We gather the required information. Our desired mean is **76**. The sum of the scores we have is given as **462**. We have 6 scores and want the mean for a total of **7** games.

We now substitute this info into our formula and solve for x:

$$Desired\ Mean = \frac{Current\ Sum\ of\ Scores + x}{Total\ Number\ of\ Scores}$$

$$76 = \frac{462 + x}{7}$$

$$532 = 462 + x$$

$$70 = x$$

Nick needs to get a 70 on his next golf game to have an average of 76

Sample Question: Sadie has an average of 84% after taking 8 chemistry tests. What grade would she need on her next test to bump her average up to 85%?

Solution: We gather the required information. Our desired mean is **85**. The sum of the current scores is 84 * 8 = **672**. We have 8 scores and want the mean for a total of **9** tests.

We now substitute this info into our formula and solve for x:

$$Desired\ Mean = \frac{Current\ Sum\ of\ Scores + x}{Total\ Number\ of\ Scores}$$

$$85 = \frac{672 + x}{9}$$

$$765 = 672 + x$$

$$93 = x$$

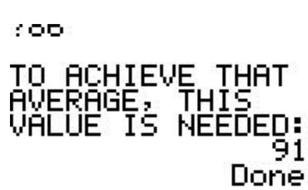
Sadie needs a 93% on her next test to obtain a new average of 85%

Now Check It Using the Calculator

Sample Question: Bob has received test grades of 84, 92, 89 and 74. What will he need on the fifth test to achieve a mean of 86?

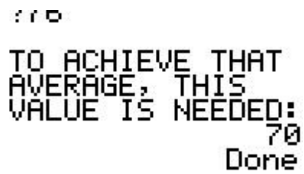
** It's best to compute the Sum of All Scores you have BEFORE you start the program!

- Open the program entitled "AVGINEED"
- Enter 4 for HOW MANY SCORES DO YOU HAVE? (This program requires you to enter the number of scores you have so far)
- Enter 339 for WHAT IS THE SUM OF ALL SCORES? (84+92+89+74=339)
- Enter 86 for WHAT IS THE DESIRED AVERAGE?
- Solution:



Sample Question: For his first 6 golf games, Nick has a total combined score of 462. What score would he need on his next golf game to have an average score of 76?

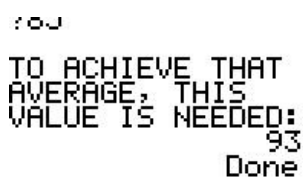
- Open the program entitled "AVGINEED"
- Enter 6 for HOW MANY SCORES DO YOU HAVE? (This program requires you to enter the number of scores you have so far)
- Enter 462 for WHAT IS THE SUM OF ALL SCORES?
- Enter 76 for WHAT IS THE DESIRED AVERAGE?
- Solution:



Sample Question: Sadie has an average of 84% after taking 8 chemistry tests. What grade would she need on her next test to bump her average up to 85%?

** It's best to compute the Sum of All Scores you have BEFORE you start the program!

- Open the program entitled "AVGINEED"
- Enter 8 for HOW MANY SCORES DO YOU HAVE? (This program requires you to enter the number of scores you have so far)
- Enter 672 for WHAT IS THE SUM OF ALL SCORES? (84 * 8 = 672)
- Enter 85 for WHAT IS THE DESIRED AVERAGE?
- Solution:



Tutorial #2

Finding the Distance Between Two Points

What Is The Distance Between Two Points? And How Do We Find It?

The distance between 2 points in the coordinate plane is a measure of how far apart they are. Or, the length of the segment connecting the 2 points.

In order to find distance, we need 2 points in the form (x_1, y_1) and (x_2, y_2).

Distance can be either a number or it can be expressed as "d" units.

The Distance Formula:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

Using the Distance Formula

Sample Question: Find the distance between the points (-1, 4) and (3, 7).

Solution: Use the distance formula with x_1= -1, y_1 = 4, x_2 = 3, and y_2 = 7

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(-1-3)^2 + (4-7)^2}$$

$$= \sqrt{(-4)^2 + (-3)^2}$$

$$= \sqrt{16+9}$$

$$= \sqrt{25}$$

$$= \mathbf{5}$$

The distance between the 2 points is 5 units

Sample Question: What is the distance between (2, -4) and (-7, 9)?

Solution: Use the distance formula with x_1= 2, y_1 =-4, x_2 = -7, and y_2 = 9

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \sqrt{(2-(-7))^2 + (-4-9)^2}$$

$$= \sqrt{(9)^2 + (-13)^2}$$

$$= \sqrt{81 + 169}$$

$$= \sqrt{250}$$

$$= \mathbf{\sqrt{25}\sqrt{10} = 5\sqrt{10} \approx 15.81}$$

The distance between the 2 points is $\boldsymbol{5\sqrt{10}\ or\ 15.81}$ units

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A Quick Note About Radicals…

The ACT varies in how they display their answers. If a radical does not simplify nicely (such as when $\sqrt{81} = 9$), it will sometimes be displayed in exact radical form and sometimes as a decimal approximation

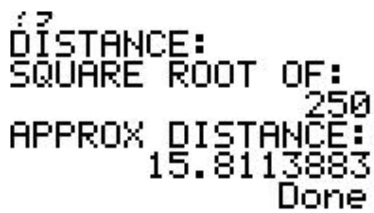
For example, $\sqrt{60}$ could be expressed as either $2\sqrt{15}$ or 7.75

Always look at the answer choices to find out which format a question expects!!!

Now Check It Using The Calculator….

Sample Question: What is the distance between (2, -4) and (-7, 9)?

- Open the program entitled "DISTANCE"
- Press ENTER to move past the opening screen
- Enter 2 for X1, -4 for Y1, -7 for X2, and 9 for Y2
- Solution:



NOTE: The calculator does not have the functionality to reduce radicals so this program gives you both the square root in unreduced form AND a decimal approximation.

Tutorial #3

Finding the Other Endpoint of a Segment When You Are Given the Midpoint and One Endpoint

What Is A Midpoint and How Do We Find One?

A Midpoint is the point which divides a line segment into two smaller, equal segments

Or…. the "halfway point" of a line segment

Given 2 points, (x_1, y_1) and (x_2, y_2), the **Midpoint Formula** is:

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Using The Midpoint Formula

Sample Question: What is the midpoint of the segment with endpoints (-3, 6) and (2, -4)?

Solution: Use the midpoint formula with x_1= -3, y_1 = 6, x_2 = 2, and y_2 = -4

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{-3+2}{2}, \frac{6+(-4)}{2}\right)$$

$$= \left(\frac{-1}{2}, \frac{2}{2}\right)$$

$$= \left(-\frac{1}{2}, 1\right)$$

But What If We Know the *Midpoint* and Need to Find the OTHER Endpoint?

- Don't panic!!! We just need to use the midpoint formula in a slightly different way
- This time, we know the coordinates of the MIDPOINT and ONE of the endpoints
- We work backwards with the formulas and it all works out just fine
- The key is knowing *WHAT YOU HAVE* and *WHAT YOU NEED*!

Using The Midpoint Formula Creatively

Sample Question: C is the midpoint of segment $\overline{AB}$. If the coordinates of point A are (-3, 4) and point C are (7, 12), what are the coordinates of point B?

Solution: This time we are given the midpoint and one endpoint. Let's assign variables to the midpoint $(x_m,\ y_m)$. In this scenario, x_m=7 and y_m=12. For our given endpoint A, x_1=-3 and y_1=4.

Again, we have x_m=7, y_m=12, x_1=-3 and y_1=4

We work with the x and y coordinates separately using the midpoint formula:

$$(x_m,\ y_m) = \left(\frac{x_1 + x_2}{2},\ \frac{y_1 + y_2}{2}\right)$$

- $x_m = \frac{x_1 + x_2}{2}$
- $7 = \frac{-3 + x_2}{2}$
- $14 = -3 + x_2$
- $17 = x_2$

- $y_m = \frac{y_1 + y_2}{2}$
- $12 = \frac{4 + y_2}{2}$
- $24 = 4 + y_2$
- $20 = y_2$

So, the coordinates of Endpoint B = (x_2, y_2) are $(17, 20)$

Sample Question: The midpoint of a segment is (-2, 4). If one endpoint is (6, -3), find the coordinates of the other endpoint.

Solution: This time we are given the midpoint and one endpoint. Let's assign variables to the midpoint (x_m, y_m). In this scenario, x_m=-2 and y_m=4. For our given endpoint, x_1=6 and y_1=-3.

Again, we have x_m=-2, y_m=4, x_1=6 and y_1=-3

We work with the x and y coordinates separately using the midpoint formula:

$$(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

- $x_m = \frac{x_1 + x_2}{2}$
- $-2 = \frac{6 + x_2}{2}$
- $-4 = 6 + x_2$
- $-10 = x_2$

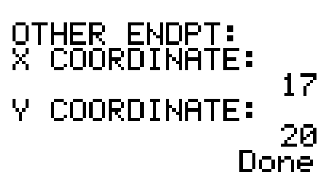
- $y_m = \frac{y_1 + y_2}{2}$
- $4 = \frac{-3 + y_2}{2}$
- $8 = -3 + y_2$
- $11 = y_2$

So, the coordinates of the Endpoint = $(\mathbf{x_2}, \mathbf{y_2})$ = $(\mathbf{-10}, \mathbf{11})$

Now Check It Using The Calculator

Sample Question: C is the midpoint of segment $\overline{AB}$. If the coordinates of point A are (-3, 4) and point C are (7, 12), what are the coordinates of point B?

- Open the program entitled "ENDPTMID"
- NOTICE THAT THE PROGRAM ASKS FOR THE ENDPOINT COORDINATES FIRST – ALWAYS MAKE SURE YOU ARE ENTERING THE RIGHT INFO!
- Enter -3 for Endpoint X, 4 for Endpoint Y, 7 for Midpoint X and 12 for Midpoint Y
- Solution:



Tutorial #4

Finding the Slope and Y-Intercept of a Line Given 2 Points

What are Slope and Y-Intercept?

- The **slope** of a line is a numerical measure of how steep the line is and in what direction it rises
- The **y-intercept** of a line is the point on the coordinate plane (or the x-y grid) where the line crosses the y-axis
- The y-intercept is often expressed as a point (0, #)

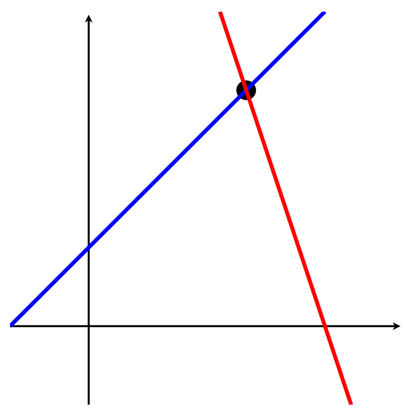


Image: Jim.belk / Public Domain

Slope-Intercept Form of a Line

A Line in Slope-Intercept Form is expressed in the form **y = mx + b**, where m and b are any real numbers and can be either positive or negative

The advantage of this form is that we can immediately tell the slope (**m**) of a line and its y-intercept (**b**)

To find the equation of a line given 2 points, we begin by finding the SLOPE and then proceed to find the Y-INTERCEPT

Finding the Slope of A Line Given 2 Points

When you are given 2 points (x_1, y_1) and (x_2, y_2) the slope formula is:

$$\boldsymbol{Slope = m = \frac{y_2 - y_1}{x_2 - x_1}}$$

Example: Find the slope of the line containing the points (-1, -5) and (3, 9)

Solution: We have $x_1 = -1$, $y_1 = -5$, $x_2 = 3$ and $y_2 = 9$

Substituting into the equation and simplifying gives:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - (-5)}{3 - (-1)} = \frac{14}{4} = \boldsymbol{\frac{7}{2}}$$

Now We Find the Y-Intercept…

Once we have the slope (m), we need to find the y-intercept (b). We do this by using the slope, one point (x,y) and the equation y= mx + b

Example: Find the y-intercept for the line containing the points (-1, -5) and (3, 9)

Solution: First we need to find the slope (m), which we found above:

$$m = \frac{7}{2}$$

We can choose ***either*** point. Let's use (3, 9)

So we have $m = \frac{7}{2}$. By choosing the point (3, 9), $x = 3$ and $y = 9$

Substitute these values into y = mx + b and solve for **b**:

$$y = mx + b$$

$$9 = \frac{7}{2}(3) + b$$

$$9 = \frac{21}{2} + b$$

$$\frac{18}{2} = \frac{21}{2} + b$$

$$-\frac{3}{2} = \boldsymbol{b}$$

Putting It All Together…

- Now that we have our slope m and our y-intercept b, we insert them into the slope-intercept form of the line: y = mx + b
- For our example, $m = \frac{7}{2}$ and $b = -\frac{3}{2}$
- Our final equation, then, is $\boldsymbol{y = \frac{7}{2}x - \frac{3}{2}}$

Note that sometimes a problem will ask for the equation. Other times, you will be asked for the slope, y-intercept or both. Be sure to read the question and answer choices carefully!

Now For Some Practice…

Sample Question: What is the equation of the line through the points (3, -1) and (6, 8) in slope intercept form?

Solution: We begin by using the slope formula with

$$x_1 = 3, y_1 = -1, x_2 = 6 \text{ and } y_2 = 8:$$

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - (-1)}{6 - 3} = \frac{9}{3} = 3$$

Now, we need to find the y-intercept (b)

We can choose either point. Let's use (3, -1). So $x = 3$ and $y = -1$

With $m = 3$, $x = 3$ and $y = -1$, we substitute into y = mx + b:

$$y = mx + b$$

$$-1 = 3(3) + b$$

$$-1 = 9 + b$$

$$\boldsymbol{-10 = b}$$

With $m = 3$ and b= -10 we substitute into the final equation:

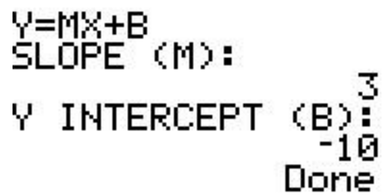
$$y = mx + b$$

$$\boldsymbol{y = 3x - 10}$$

Now Check It Using The Calculator….

Sample Question: What is the equation of the line through the points (3, -1) and (6, 8) in slope intercept form?

- Open the program entitled "EQOFLINE"
- Enter 3 for X1, -1 for Y1, 6 for X2 and 8 for Y2
- Solution:



Tutorial #5

Solving Percentage Problems

The Key to Solving Percentage Problems:

- SO many students struggle with problems like "75 is what percent of 300?"
- The goal is to SIMPLIFY the problems and make them easy to solve
- We do this by ***CONVERTING*** the words into a mathematical equation
- Then the solution becomes simple

So What Are We Looking For?

The process is like a treasure hunt… We are looking for three key pieces of information:

- What?
- Is?
- Of?

If we can identify these values, the rest is simple…

Now we determine how to change the words into symbols:

1) What? is represented by the variable, usually **x**
2) Is? means **=** (equals)
3) Of? means we **multiply**

Before we begin one key note about **percentages**…

Expressing Percentages as Decimals (and Vice Versa)

To convert a percentage to a decimal, we *divide it by 100:*

For example: 75% is equivalent to $\frac{75}{100} = .75$

143% is equivalent to $\frac{143}{100} = 1.43$

To change a decimal to a percent, we *multiply it by 100:*

For example: .634 is equivalent to $.634 * 100 = 63.4\%$

1.4 is equivalent to $1.4 * 100 = 140\%$

Using the Formulas:

Sample Question: What number is 60% of 75?

Solution: Identify the key pieces of information and match them up with their mathematical symbols:

What number is 60% of 75?

x = .60 * 75

x = 45

Sample Question: 75 is what percent of 300?

Solution: Identify the key pieces of information and match them up with their mathematical symbols:

75 is what percent of 300?

75 = x * 300

75 = 300x

x = .25

We convert .25 to a percentage by multiplying by 100: **.25 * 100 = 25%**

Sample Question: 42 is 40% of what number?

Solution: Identify the key pieces of information and match them up with their mathematical symbols:

42 is 40% of what number?

42 = .40 * x

42 = .40x

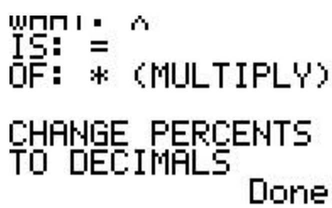
105 = x

How The Calculator Can Help…

While we don't have a program to do the calculations for you, we still provide you with a reminder of the key words and symbols:

- Open the program entitled "EQSYMBOL"

- Press ENTER to see the final reminder:



Tutorial #6

Finding the Number of Sides in a Polygon Given the Measures of Angles

What Is A Polygon?

- **A polygon is a two-dimensional figure that has 3 or more straight sides**

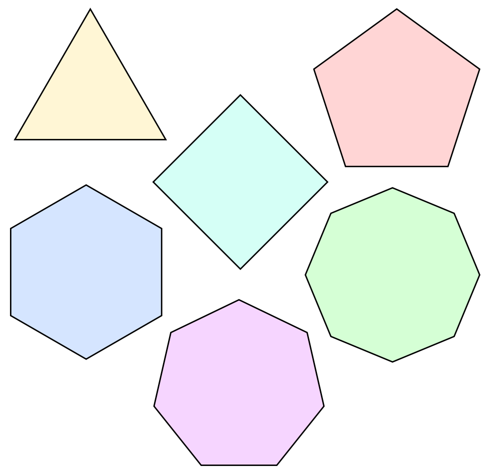


Image: Quartl / CC BY-SA

- **Some common examples are triangles, quadrilaterals and pentagons, though a polygon can have many, many sides**

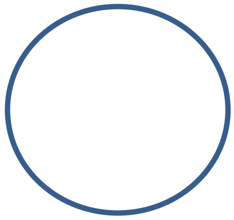
Names of Common Polygons

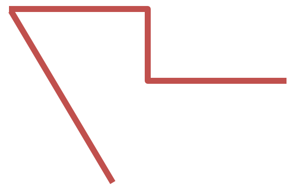
Number of Sides (n)	Name	Number of Sides (n)	Name
3	Triangle	8	Octagon
4	Quadrilateral	9	Nonagon
5	Pentagon	10	Decagon
6	Hexagon	12	Dodecagon
7	Heptagon	n not listed here	n-gon*

* For example, a 15 sided figure is commonly called a 15-gon

What is NOT a Polygon?

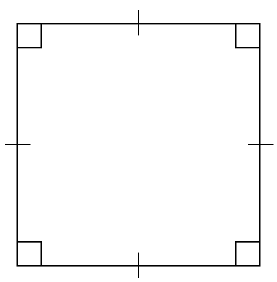
› It is equally important to know what is NOT a polygon: Any figure with curved sides (such as a circle) or one or more sides that don't connect.





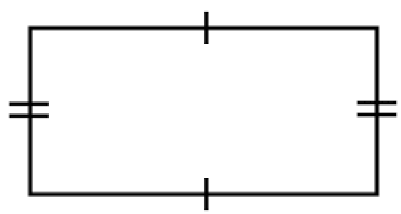
Regular vs. Irregular Polygons

› A **regular polygon** is a polygon whose sides are all the same length and angles are all the same measure. For example, a square IS a regular polygon but a rectangle is not.



Regular Polygon

Image: Enoshd / CC BY-SA 4.0



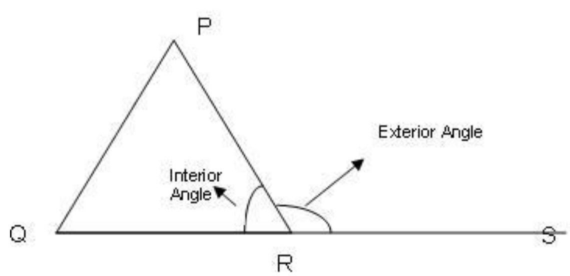
Irregular Polygon

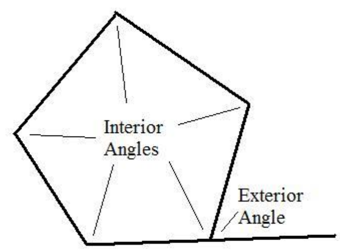
Image: Enoshd / CC BY-SA 4.0

› **It is important to distinguish between regular and irregular polygons when we want to find the measures of individual angles in polygons!**

Interior vs. Exterior Angles

- Interior angles are usually easy to identify, but students sometimes are confused about where exterior angles are.
- It is important to note that exterior angles are not always visible. The sides of polygons need to be extended to see them, and some figures will not have the sides extended.
- However, exterior angles always exist and their measures can be calculated!





Formulas: Interior Angles in a Polygon

- For all polygons, $n = number\ of\ sides$
- As long as the polygon has 3 or more sides, we can find the **SUM OF INTERIOR ANGLES** whether it is regular or not:

$$Sum_{Interior} = 180(n-2)$$

- If the polygon is **REGULAR**, we can also find the measure of ONE interior angle in the polygon:

$$One_{Interior} = \frac{180(n-2)}{n}$$

- **Do NOT use this formula if you are not told the polygon is regular!!!**

Formulas: Exterior Angles in a Polygon

- For all polygons, $n = number\ of\ sides$
- No matter how many sides a polygon has (and whether it is regular or not) the **SUM OF EXTERIOR ANGLES** is always the same:

$$Sum_{Exterior} = 360°$$

- If the polygon is **REGULAR**, we can also find the measure of ONE exterior angle in the polygon:

$$One_{Exterior} = \frac{360}{n}$$

- **Do NOT use this formula if you are not told the polygon is regular!!!**

But What If I Don't Know The Number Of Sides???

- This is not a problem! Sometimes ACT problems give us information about one or more angles in the polygons, and we have to work backwards to find the number of sides, n
- It is important to know that we can find n given any one of 3 different pieces of information:

   1. If we know the SUM of the measures of INTERIOR angles
   2. If we know the measure of ONE INTERIOR angle in a regular polygon
   3. If we know the measure of ONE EXTERIOR angle in a regular polygon

- **Since the SUM of the measures of the EXTERIOR angles of <u>ANY</u> polygon is 360, knowing this information does not help us at all!**

Finding The Number of Sides of a Polygon When You Know the Sum of the Interior Angles

Sample Question: The sum of the interior angles of a polygon is 1980°. How many sides does the polygon have?

Solution: We work backwards using the Sum Interior formula:

$$Sum_{Interior} = 180(n-2)$$

$$1980 = 180(n-2)$$

$$1980 = 180n - 360$$

$$2340 = 180n$$

$$13 = n$$

The polygon whose sum of interior angles is 1980° has 13 sides.

Finding the Number of Sides When You Know the Measure of One Interior Angle in a Regular Polygon

Sample Question: One interior angle of a regular polygon measures 144°. What is the name of the polygon?

Solution: * We may only proceed since the polygon is REGULAR! *

We work backwards using the One Interior Formula:

$$One_{Interior} = \frac{180(n-2)}{n}$$

$$144 = \frac{180(n-2)}{n}$$ *Cross-Multiply!*

$$144n = 180(n-2)$$

$$144n = 180n - 360$$

$$-36n = -360$$

$n = 10$ ⇨ *Decagon*

The regular polygon with one interior angle measuring 144° is a decagon (a 10-sided polygon)

Finding the Number of Sides When You Know the Measure of One Exterior Angle in a Regular Polygon

Sample Question: A regular polygon has exterior angles which each measure 45°. How many sides does the polygon have?

Solution: * We may only proceed since the polygon is REGULAR! *

We work backwards using the One Exterior Formula:

$$One_{Exterior} = \frac{360}{n}$$

$$45 = \frac{360}{n} \qquad \textit{Cross-Multiply!}$$

$$45n = 360$$

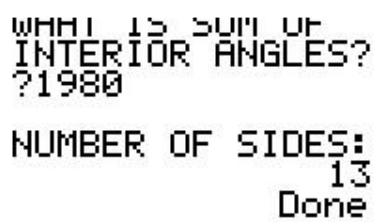
$$n = 8$$

The regular polygon with one exterior angle measuring 45° has 8 sides.

Now Check It Using The Calculator

Sample Question: The sum of the interior angles of a polygon is 1980°. How many sides does the polygon have?

- Open the program entitled “FINDN”
- Press 1 for SUM INTERIOR
- Enter 1980 for What Is Sum Of Interior Angles?



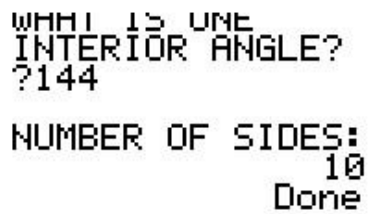
Sample Question: One interior angle of a regular polygon measures 144°. What is the name of the polygon?

› Open the program entitled "FINDN"

› Press 2 for ONE INTERIOR

› Enter 144 for What Is One Interior Angle?

› Solution:



› **Important: The calculator just gives you the number of sides. You still need to know that an 10-sided polygon is called a DECAGON!**

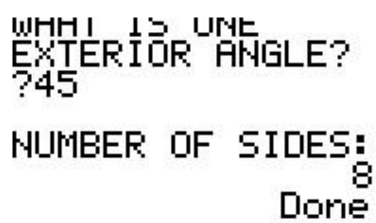
Sample Question: A regular polygon has exterior angles which each measure 45°. How many sides does the polygon have?

› Open the program entitled "FINDN"

› Press 3 for ONE EXTERIOR

› Enter 45 for What Is One Exterior Angle?

› Solution:



Tutorial #7

Finding the Greatest Common Factor (GCF)

What Is A Greatest Common Factor (GCF)?

The GCF of two numbers is the largest number that divides evenly into those two numbers. For example, the GCF for 50 and 20 is 10.

It is also possible to find the GCF of variable expressions:

For example, the GCF for x^3y and x^2y^3 is x^2y

Finally, we can find the GCF for expressions that have both numbers and variables.

For example, the GCF for $27a^3b^2$ and $18ab^3$ is $9ab^2$

What is a Prime Factorization?

The prime factorization of a number is found by expressing a number as a series of prime numbers multiplied together.

A prime number is any number which is only divisible by itself and 1. Basically, any number which can be divided by another integer (that isn't 1) is NOT PRIME. **The first prime number is 2 (not 1!).**

Here are the first 10 prime numbers: **2, 3, 5, 7, 11, 13, 17, 19, 23, 29**

How to Find the GCF for 2 Numbers

There are multiple ways to find the GCF of 2 numbers. In this lesson, we will begin by writing out the prime factorizations of the 2 numbers.

- Note: There are multiple ways to write the factorization for a number and as long as you reduce completely, you will always end up with the same prime factorization.

For example, two ways to find the SAME prime factorization of 54 are:

$$54 = 2 * 27 = 2 * 3^3 = 2 * 3 * 3 * 3$$

$$54 = 6 * 9 = 2 * 3 * 3^2 = 2 * 3 * 3 * 3$$

Sample Problem: Find the GCF for 54 and 120

Begin by expanding each of the numbers:

$$54 = 2 * 27 = 2 * 3^3 = 2 * 3 * 3 * 3$$

$$120 = 8 * 15 = 2^3 * 3 * 5 = 2 * 2 * 2 * 3 * 5$$

Now, to find the GCF, highlight any factors that exist in both prime factorizations (multiples are ok if they appear in both numbers):

$$54 = 2 * 3 * 3 * 3$$

$$120 = 2 * 2 * 2 * 3 * 5$$

The last step is to MULTIPLY the shared factors:

The GCF for 54 and 120 is $\mathbf{2 * 3 = 6}$

What If There Are Variables Involved? No Problem!!!

If an expression contains variables, simply write out each power of the variable(s) as its own factor. For example, we would write the factorization of x^4 as $x * x * x * x$

If numbers and variables are combined together, it is still not difficult. The prime factorization of $42a^3b^2$ is:

$$\mathbf{42 * a * a * a * b * b}$$

$$\mathbf{= 6 * 7 * a * a * a * b * b}$$

$$\mathbf{= 2 * 3 * 7 * a * a * a * b * b}$$

How to Find the GCF For 3 Numbers

Sometimes you will be asked to find the Greatest Common Factor for 3 Numbers. There are 2 ways to accomplish this:

A) Write out the prime factorizations for all of the numbers and then find shared factors for ALL 3 numbers

B) Find the GCF for the first 2 numbers. Then take that GCF and the 3rd number, and find the GCF for those 2 numbers

The next 2 examples show the same problem solved in each of these different ways. You can choose which one you prefer!

Finding the GCF for 3 Numbers – Method A

Sample Problem: Find the GCF for 78, 84 and 135

Solution: Begin by expanding each of the numbers:

$$78 = 2 * 39 = 2 * 3 * 13$$

$$84 = 2 * 42 = 2 * 6 * 7 = 2 * 2 * 3 * 7$$

$$135 = 27 * 5 = 3^3 * 5 = 3 * 3 * 3 * 5$$

Now, to find the GCF, highlight any factors that exist in all 3 prime factorizations (multiples are ok if they appear in all 3 numbers)

$$78 = 2 * 3 * 13$$

$$84 = 2 * 2 * 3 * 7$$

$$135 = 3 * 3 * 3 * 5$$

3 is the only shared factor for all three numbers

Therefore, the GCF for 78, 84 and 135 is 3

Finding the GCF for 3 Numbers – Method B

Sample Problem: Find the GCF for 78, 84 and 135

Solution: Step One: Find the GCF for the first 2 numbers by highlighting any factors that exist in both prime factorizations:

$$78 = \mathbf{2} * \mathbf{3} * \mathbf{13}$$

$$84 = \mathbf{2} * \mathbf{2} * \mathbf{3} * \mathbf{7}$$

The GCF for the first 2 numbers is $\mathbf{2} * \mathbf{3} = \mathbf{6}$

Step Two: Now find the GCF for 6 and 135:

$$6 = \mathbf{2} * \mathbf{3}$$

$$135 = \mathbf{3} * \mathbf{3} * \mathbf{3} * \mathbf{5}$$

The GCF for these 2 numbers is 3

Therefore, the GCF for 78, 84 and 135 is 3

Now For Some Practice…

Sample Question: What is the greatest common factor of 64 and 120?

Solution: Begin by finding the prime factorizations for each number:

$64 = 16 * 4 = 4 * 4 * 4 = 2 * 2 * 2 * 2 * 2 * 2$

$120 = 8 * 15 = 2^3 * 3 * 5 = 2 * 2 * 2 * 3 * 5$

Highlight the shared factors:

$64 = 2 * 2 * 2 * 2 * 2 * 2$

$120 = 2 * 2 * 2 * 3 * 5$

Now multiply the shared factors: $2 * 2 * 2 = 8$

The GCF for 64 and 120 is $\mathbf{8}$

Sample Question: What is the greatest common factor for $32x^2$ and $52x^5$?

Solution: Begin by finding the prime factorizations for each expression:

$32x^2 = 8 * 4 * x^2 = 2 * 4 * 4 * x * x = 2 * 2 * 2 * 2 * 2 * x * x$

$52x^5 = 2 * 26 * x^5 = 2 * 2 * 13 * x * x * x * x * x$

Highlight the shared factors:

$32x^2 = 2 * 2 * 2 * 2 * 2 * x * x$

$52x^5 = 2 * 2 * 13 * x * x * x * x * x$

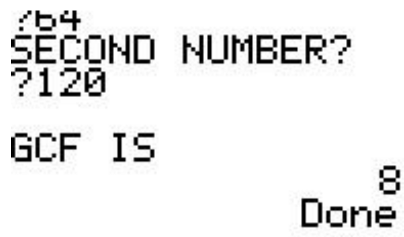
Now multiply the shared factors: $2 * 2 * x * x = 4x^2$

The GCF for the two expressions is $\mathbf{4x^2}$

Now Check It Using The Calculator…

Sample Question: What is the greatest common factor of 64 and 120?

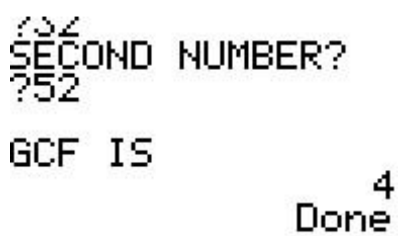
- Open the program entitled "GCF"
- Enter 64 for "FIRST NUMBER" and 120 for "SECOND NUMBER"
- Solution:



Sample Question: What is the greatest common factor for $32x^2$ and $52x^5$?

Note: The calculator will NOT be able to help you find the GCF for variable expressions. You can find the numerical GCF but still have to use algebra to find the GCF for the variable portion.

- Open the program entitled "GCF"
- Enter 32 for "FIRST NUMBER" and 52 for "SECOND NUMBER"
- Solution:



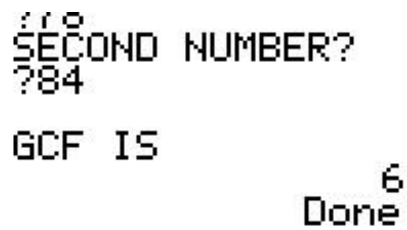
The GCF for the numerical portion is **4**. The GCF for the variable portion is x^2

Therefore, the GCF for both expressions is $4x^2$

Sample Question: **What is the greatest common factor of 78, 84 and 135?**

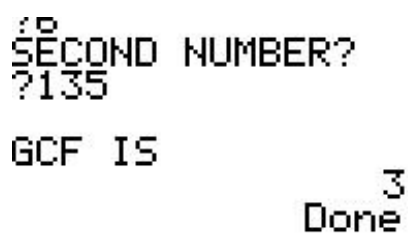
Note: The calculator can only find the GCF for 2 numbers at a time, so we have to use the 2-step process described in Method B

- Open the program entitled "GCF"
- Enter 78 for "FIRST NUMBER" and 84 for "SECOND NUMBER"
- Solution:



- Press ENTER to run the program again

- Enter 6 for "FIRST NUMBER" and 135 for "SECOND NUMBER"
- Solution:



Tutorial #8

Solving Percentage of Increase/Decrease Word Problems

We encounter percentage of increase/decrease problems very often in daily life: purchases at a store, leaving tips, calculating sales tax, etc…

- The ACT will ask you questions about markdowns, increasing a price by a certain percentage and other similar problems
- The key is properly converting percentages to decimals AND using the correct formulas

---

Expressing Percentages as Decimals

To convert a percentage to a decimal, we *divide it by 100:*

For example: 75% is equivalent to $\frac{75}{100} = .75$

143% is equivalent to $\frac{143}{100} = 1.43$

Another way to think about it is *moving the decimal point 2 spaces to the left <u>and adding zeros as needed</u>:*

For example: 247% = 247.% = 2.47

5% = 5.% = .05

Percentage Increase/Decrease Formulas

The Variables: A = New Amount

P = Original Amount

r = Interest Rate ***<u>as a decimal!</u> This is very important!!***

- **Percentage Increase: A = P (1 + r)**
- **Percentage Decrease:A = P (1 – r)**

Using the Formulas

Sample Question: A customer wants to leave an 18% tip on a dinner bill in the amount of $75. How much total money should he leave?

Solution: Use the Percentage Increase Formula with $r = .18$ and $P = 75$:

$$A = P\,(1 + r)$$

$$A = 75\,(1 + .18)$$

$$A = 75\,(1.18)$$

$$A = 88.5 \text{ or } \$88.50$$

The customer should leave a total of $88.50

Sample Question: Rebecca used a 30% coupon to purchase a television that was originally priced at $699. How much did she pay?

Solution: Use the Percentage Decrease Formula with $r = .30$ and $P = 699$:

$$A = P\,(1 - r)$$

$$A = 699\,(1 - .30)$$

$$A = 699\,(.70)$$

$$A = 489.30 \text{ or } \$489.30$$

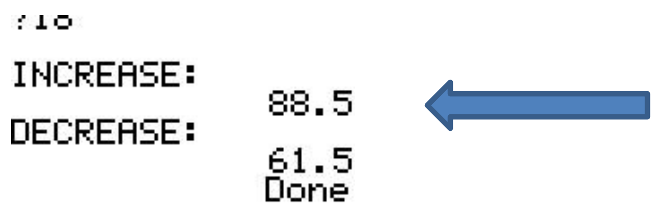
Rebecca paid $489.30 for the television

Now Check It Using the Calculator

Sample Question: A customer wants to leave an 18% tip on a dinner bill in the amount of $75. How much total money should he leave?

- Open the program entitled “INCRDECR”
- Enter 75 for ORIGINAL AMOUNT and 18 for PERCENT CHANGE

  (Note that we enter the percentage ***WITHOUT*** converting to a decimal!)
- Solution:



Sample Question: Rebecca used a 30% coupon to purchase a television that was originally priced at $699. How much did she pay?

- Open the program entitled “INCRDECR”
- Enter 699 for ORIGINAL AMOUNT and 30 for PERCENT CHANGE

  (Note that we enter the percentage ***WITHOUT*** converting to a decimal!)
- Solution:

Tutorial #9

Finding the Interior and Exterior Angles of a Polygon

What Is A Polygon?

› **A polygon is a two-dimensional figure that has 3 or more straight sides**

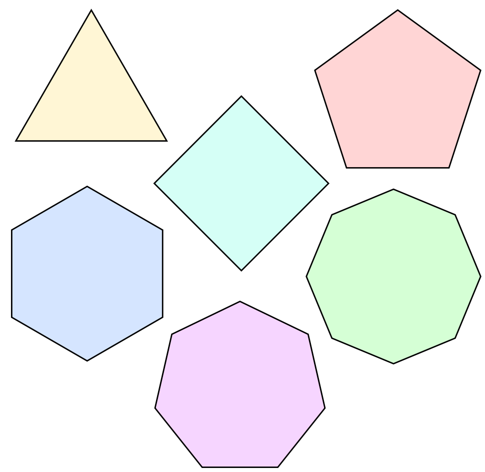


Image: Quartl / CC BY-SA

› **Some common examples are triangles, quadrilaterals and pentagons, though a polygon can have many, many sides**

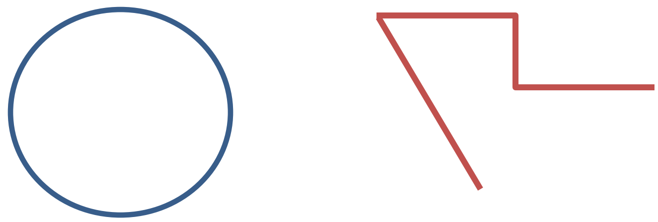
Names of Common Polygons

Number of Sides (n)	Name	Number of Sides (n)	Name
3	Triangle	8	Octagon
4	Quadrilateral	9	Nonagon
5	Pentagon	10	Decagon
6	Hexagon	12	Dodecagon
7	Heptagon	n not listed here	n-gon*

* For example, a 15 sided figure is commonly called a 15-gon

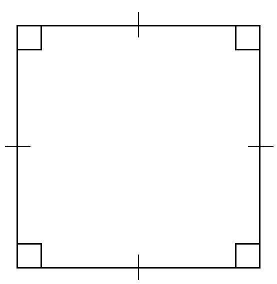
What is NOT a Polygon?

- It is equally important to know what is NOT a polygon: Any figure with curved sides (such as a circle) or one or more sides that don't connect.



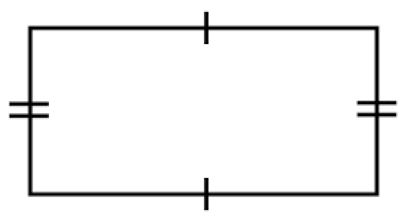
Regular vs. Irregular Polygons

- A **regular polygon** is a polygon whose sides are all the same length and angles are all the same measure. For example, a square IS a regular polygon but a rectangle is not.



Regular Polygon

Image: Enoshd / CC BY-SA 4.0



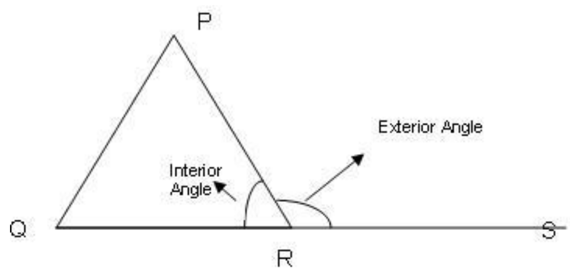
Irregular Polygon

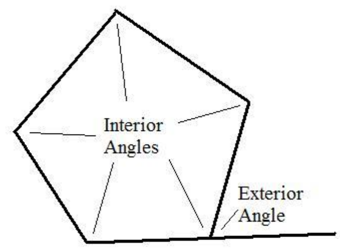
Image: Enoshd / CC BY-SA 4.0

- **It is important to distinguish between regular and irregular polygons when we want to find the measures of individual angles in polygons!**

Interior vs. Exterior Angles

- Interior angles are usually easy to identify, but students sometimes are confused about where exterior angles are.
- It is important to note that exterior angles are not always visible. The sides of polygons need to be extended to see them, and some figures will not have the sides extended.
- However, exterior angles always exist and their measures can be calculated!





Formulas: Interior Angles in a Polygon

- For all polygons, $n = number\ of\ sides$
- As long as the polygon has 3 or more sides, we can find the **SUM OF INTERIOR ANGLES** whether it is regular or not:

$$Sum_{Interior} = 180(n-2)$$

- If the polygon is **REGULAR**, we can also find the measure of ONE interior angle in the polygon:

$$One_{Interior} = \frac{180(n-2)}{n}$$

- **Do NOT use this formula if you are not told the polygon is regular!!!**

Formulas: Exterior Angles in a Polygon

- For all polygons, $n = number\ of\ sides$
- No matter how many sides a polygon has (and whether it is regular or not) the **SUM OF EXTERIOR ANGLES** is always the same:

$$Sum_{Exterior} = 360°$$

- If the polygon is **REGULAR**, we can also find the measure of ONE exterior angle in the polygon:

$$One_{Exterior} = \frac{360}{n}$$

- **Do NOT use this formula if you are not told the polygon is regular!!!**

Finding the Sum of Interior Angles in a Polygon

Sample Question: What is the sum of the measures of the interior angles for a dodecagon?

Solution: A dodecagon is a polygon with 12 sides, so n=12. It does NOT matter whether the polygon is regular or not to find the sum of interior angles:

$$Sum_{Interior} = 180(n - 2)$$

$$Sum_{Interior} = 180(12 - 2)$$

$$Sum_{Interior} = 180(10)$$

$$Sum_{Interior} = 1800°$$

The sum of the measures of the interior angles for a dodecagon is 1800°

Finding One Interior Angle in a Regular Polygon

Sample Question: What is the measure of one interior angle in a regular octagon?

Solution: An octagon has 8 sides, so n=8. We are told the polygon is regular, so we may use the One Interior Angle formula:

$$One_{Interior} = \frac{180(n-2)}{n}$$

$$One_{Interior} = \frac{180(8-2)}{8}$$

$$One_{Interior} = \frac{180(6)}{8} = \frac{1080}{8} = 135°$$

One interior angle of a regular octagon is 135°

Finding the Sum of Exterior Angles in a Polygon

Sample Question: What is the sum of the measures of the exterior angles for an 11-sided polygon?

Solution: This polygon has 11 sides, so n=11. It does NOT matter whether the polygon is regular or not to find the sum of exterior angles: $Sum_{Exterior} = 360°$

Note that it DOES NOT MATTER how many sides a polygon has! The sum of the exterior angles for ANY polygon is <u>ALWAYS 360</u>°!!!

Finding One Exterior Angle in a Regular Polygon

Sample Question: What is the measure of one exterior angle in a regular 10-sided polygon?

Solution: This polygon has 10 sides, so n=10. We are told the polygon is regular, so we may use the One Exterior Angle formula:

$$One_{Exterior} = \frac{360}{n}$$

$$One_{Exterior} = \frac{360}{10}$$

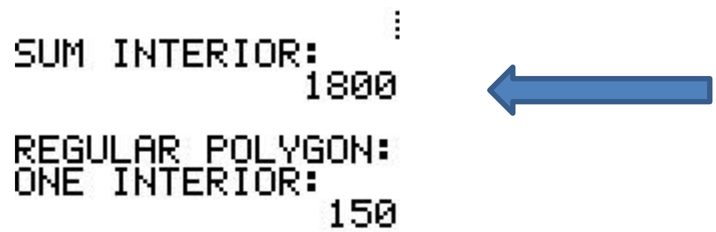
$$One_{Exterior} = 36°$$

One exterior angle of a regular 10-sided polygon is 36°

Now Check It Using the Calculator

Sample Question: What is the sum of the measures of the interior angles for a dodecagon?

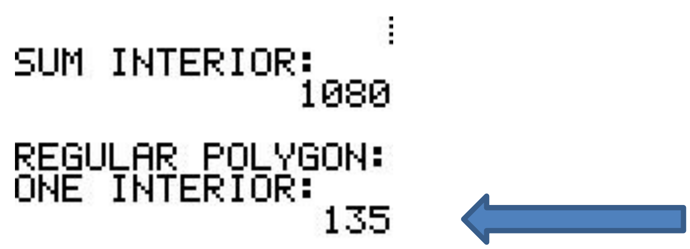
- Open the program entitled "INTEXT"
- Enter **12** for HOW MANY SIDES
- Solution:



- Press ENTER to display remaining data and end program

Sample Question: What is the measure of one interior angle in a regular octagon?

- Open the program entitled "INTEXT"
- Enter **8** for HOW MANY SIDES
- Solution:



- Press ENTER to display remaining data and end program

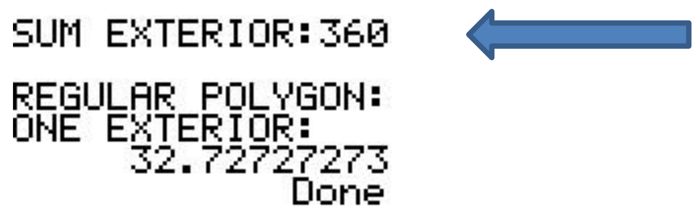
Sample Question: What is the sum of the measures of the exterior angles for an 11-sided polygon?

› Open the program entitled "INTEXT"

› Enter **11** for HOW MANY SIDES

› Press ENTER to move to next screen

› Solution:



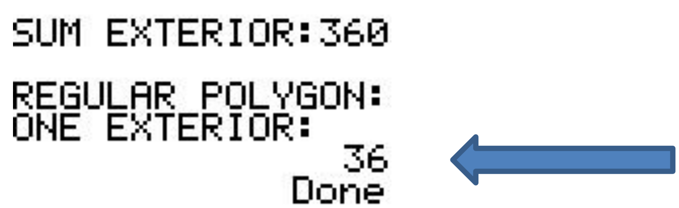
Sample Question: What is the measure of one exterior angle in a regular 10-sided polygon?

› Open the program entitled "INTEXT"

› Enter **10** for HOW MANY SIDES

› Press ENTER to move to next screen

› Solution:



Tutorial #10

Finding the Slope and Y-Intercept of a Line in Standard Form

What are Slope and Y-Intercept?

- The **slope** of a line is a numerical measure of how steep the line is and in what direction it rises.
- The **y-intercept** of a line is the point on the coordinate plane (or the x-y grid) where the line crosses the y-axis.
- The y-intercept is often expressed as a point (0, #)

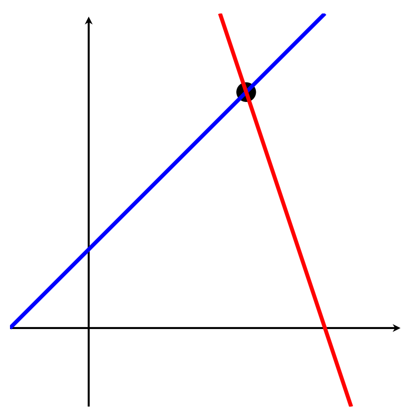


Image: Jim.belk / Public Domain

Equations of Lines

A Line in Standard Form is expressed in the form **Ax + By = C**, where A, B and are integers (no decimals or fractions) and A is usually positive

- The standard form equation does not immediately tell us the slope or y-intercept (though we can use algebra to find them)

A Line in Slope-Intercept Form is expressed in the form **y = mx + b**, where m and b are any real numbers and can be either positive or negative

- The advantage of this form is that we can immediately tell the slope (**m**) of a line and its y-intercept (**b**)

How Do I Convert A Line in Standard Form into Slope-Intercept Form?

- Simple! We use algebra! The goal is to solve the equation for **y**
- Here's a sample problem: Convert the equation $3x - 6y = 15$ into slope-intercept form:

$$3x - 6y = 15$$

$$-6y = -3x + 15$$

$$y = \frac{-3}{-6}x + \frac{15}{-6}$$

$$\boldsymbol{y = \frac{1}{2}\mathbf{x} - \frac{5}{2}}$$

So the Line is in Slope-Intercept Form… Now What?

$$y = mx + b$$

$$\boldsymbol{y = \frac{1}{2}\mathbf{x} - \frac{5}{2}}$$

- The slope (m) of this line is $\frac{1}{2}$
- The y-intercept (b) of this line is $-\frac{5}{2}$
- Expressed as a point, the y-intercept is $(0, \ -\frac{5}{2})$

Another Way to Find Intercepts

- The methods we just explained allow you to find the slope and y-intercept of a line by converting it from standard form to slope-intercept form
- There is another way to find the y-intercept (and also the x-intercept) without converting the equation. This can be faster if you are not also asked for the slope
- To find our y-intercept, we use the fact that the y-intercept of a line is the point where it crosses the y axis. **At this point, x will equal 0!**
- To find our x-intercept, we use the fact that the x-intercept of a line is the point where it crosses the x axis. **At this point, y will equal 0!**

Sample Problem: Find the x- and y-intercepts for the line $3x - 7y = 18$

To Find the X-Intercept: The x-intercept is the value of x when y=0. Substitute 0 for y, and solve for x:

$$3x - 7y = 18$$

$$3x - 7(0) = 18$$

$$3x - 0 = 18$$

$$3x = 18$$

$$x = 6$$

The x-intercept is 6. Expressed as a point, the x-intercept is (6, 0)

To Find the Y-Intercept: The y-intercept is the value of y when x=0. Substitute 0 for x, and solve for y:

$$3x - 7y = 18$$

$$3(0) - 7y = 18$$

$$0 - 7y = 18$$

$$-7y = 18$$

$$y = -\frac{18}{7}$$

The y-intercept is $-\frac{18}{7}$ Expressed as a point, the x-intercept is (0,$-\frac{18}{7}$)

Now For Some More Practice:

Sample Question: What is the y-intercept of the line $-4x + 6y = 30$?

Solution: Use algebra to convert the equation into y = mx + b form:

$$-4x + 6y = 30$$

$$6y = 4x + 30$$

$$y = \frac{4}{6}x + \frac{30}{6}$$

$$\boldsymbol{y = \frac{2}{3}x + 5}$$

The problem asked for the y-intercept (b), which equals 5. Expressed as a point, the y-intercept is (0, 5)

Sample Question: What is the x-intercept of the line $5x - 12y = -30$?

Solution: Converting the equation to y = mx + b form will not help us find this!

Instead, we substitute 0 in for y and solve for x:

$$5x - 12y = -30$$

$$5x - 12(0) = -30$$

$$5x - 0 = -30$$

$$5x = -30$$

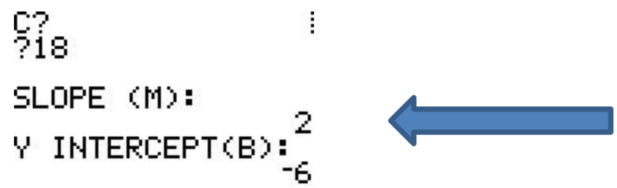
$$x = -6$$

The x-intercept equals -6. Expressed as a point, the x-intercept is (-6,0)

Now Check It Using the Calculator

Sample Question: What is the slope of the line $6x - 3y = 18$?

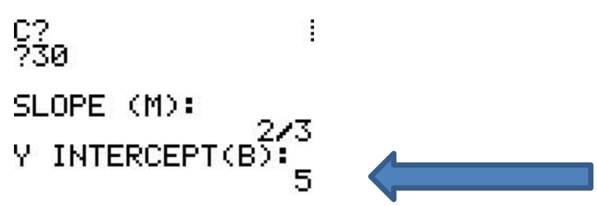
- Open the program entitled "MANDB"
- Enter 6 for A, -3 for B and 18 for C



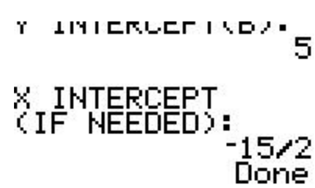
- Press ENTER to advance to the next screen. The x-intercept is displayed if needed

Sample Question: What is the y-intercept of the line $-4x + 6y = 30$?

- Open the program entitled "MANDB"
- Enter -4 for A, 6 for B and 30 for C

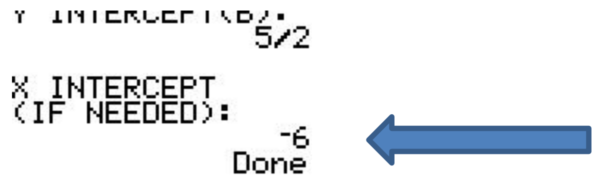


- Press ENTER to advance to the next screen. The x-intercept is displayed if needed



Sample Question: What is the x-intercept of the line $5x - 12y = -30$?

- Open the program entitled “MANDB”
- Enter 5 for A, -12 for B and -30 for C
- Press ENTER to advance to the next screen



Tutorial #11

Finding A Midpoint

What Is A Midpoint? And How Do We Find One?

- The point which divides a line segment into two smaller, equal segments
- Or…. the "halfway point" of a line segment

Given 2 points, (x_1, y_1) and (x_2, y_2), the **Midpoint Formula** is:

$$\left(\frac{\boldsymbol{x_1 + x_2}}{\mathbf{2}}, \frac{\boldsymbol{y_1 + y_2}}{\mathbf{2}}\right)$$

Using the Midpoint Formula

Sample Question: What is the midpoint of the segment with endpoints (-3, 6) and (2, -4)?

Solution: Use the midpoint formula with x_1= -3, $y_1 = 6$, $x_2 = 2$, and $y_2 =$ -4

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{-3+2}{2}, \frac{6+(-4)}{2}\right)$$

$$= \left(\frac{-1}{2}, \frac{2}{2}\right)$$

$$= \left(-\frac{1}{2}, 1\right)$$

Sample Question: R is the midpoint of segment $\overline{ST}$. If the coordinates of point S are (-2, 7) and point T are (0, -5), what are the coordinates of point R?

Solution: Use the midpoint formula with x_1= -2, y_1 = 7, x_2 = 0, and y_2 = -5

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2},\ \frac{y_1 + y_2}{2}\right)$$

$$= \left(\frac{-2+0}{2},\ \frac{7+(-5)}{2}\right)$$

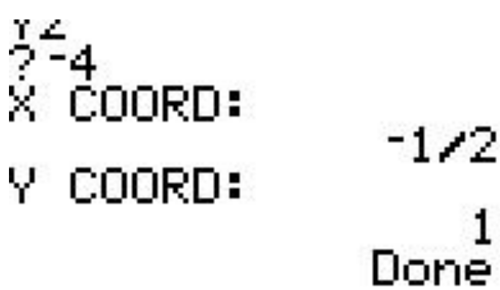
$$= \left(\frac{-2}{2},\ \frac{2}{2}\right)$$

$$= (-1,\ 1)$$

Now Check It Using The Calculator….

Sample Question: What is the midpoint of the segment with endpoints (-3, 6) and (2, -4)?

- Open the program entitled "MIDPOINT"
- Enter -3 for X1, 6 for Y1, 2 for X2, and -4 for Y2
- Solution:

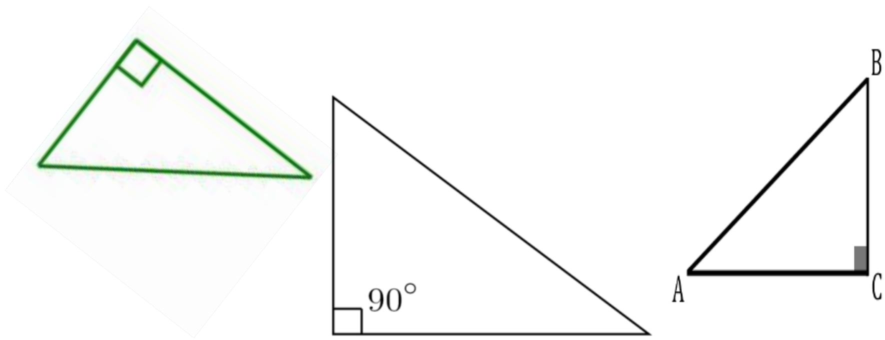


Tutorial #12

Finding the Sides of a Right Triangle

What Is A Right Triangle?

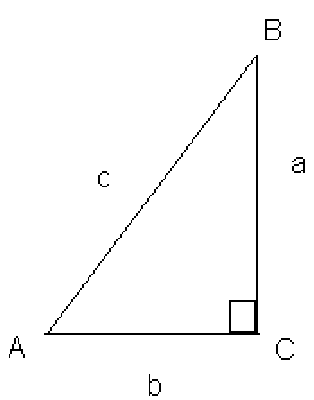
- **A right triangle is a 3-sided polygon that contains a right (90°) angle. Here are some examples:**



The Pythagorean Theorem

- The **Pythagorean Theorem** is one of math's most elegant and frequently used theorems. It describes the relationship between the 3 sides of a **right** triangle:

$$\boldsymbol{a^2 + b^2 = c^2}$$



Note: Sides a and b are the LEGS of the triangle. Side c is the HYPOTENUSE of the triangle, the side opposite (across from) the right angle. It does not matter which leg is assigned as a or b. All that matters is that you correctly label the hypotenuse

Finding the Missing Side of a Right Triangle

There are 2 basic types of questions involving sides of right triangles:

1. Find the hypotenuse when you are given the 2 legs

2. Find a leg when you are given the other leg and the hypotenuse

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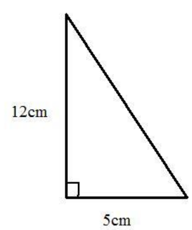
Using The Pythagorean Theorem (1)

Sample Question: The legs of a right triangle have lengths 5 cm and 12 cm. What is the length of the hypotenuse?



Solution: Since we have both legs, assign a=5 and b=12. Note that they could be interchanged. We now solve for the hypotenuse c:

$$a^2 + b^2 = c^2$$

$$5^2 + 12^2 = c^2$$

$$25 + 144 = c^2$$

$$169 = c^2$$

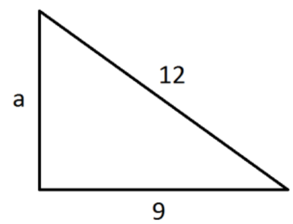
$$\sqrt{169} = \sqrt{c^2}$$

$$13 = c$$

The length of the hypotenuse is 13 cm

Using The Pythagorean Theorem (2)

Sample Question: The hypotenuse of a right triangle is 12 units. One leg is 9 units. Find the other leg.



Solution: We have the hypotenuse, so c=12. One leg is 9, so assign either a or b as 9. Looking at the diagram, it makes more sense to let b=9. We now solve for the remaining leg a:

$$\boldsymbol{a^2 + b^2 = c^2}$$

$$\boldsymbol{a^2 + 9^2 = 12^2}$$

$$\mathbf{a^2 + 81 = 144}$$

$$\boldsymbol{a^2 = 63}$$

$$\boldsymbol{\sqrt{a^2} = \sqrt{63}}$$

$$\mathbf{a} = \boldsymbol{\sqrt{9}\sqrt{7} = 3\sqrt{7} \approx 7.94}$$

The second leg is $3\sqrt{7}$, or approximately 7.94 units

A Quick Note About Radicals…

- The ACT varies in how they display their answers. If a radical does not simplify nicely (such as when $\sqrt{81} = 9$), it will sometimes be displayed in exact radical form and sometimes as a decimal approximation.
- For example, $\sqrt{60}$ could be expressed as either $2\sqrt{15}$ ***or*** 7.75.

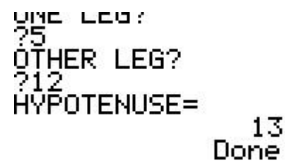
Always look at the answer choices to find out which format a question expects!!!

Now Check It Using The Calculator (1)…

Sample Question: The legs of a right triangle have lengths 5 cm and 12 cm. What is the length of the hypotenuse?

- Open the program entitled "PYTHAG"

- Enter **2** to find the HYPOTENUSE
- Enter **5** for ONE LEG and **12** for OTHER LEG
- Solution:



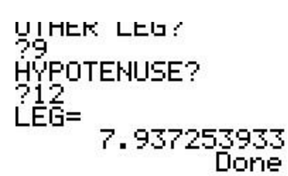
- **NOTE:** The calculator does not have the functionality to reduce radicals so it will always display decimal answers if needed.

Now Check It Using The Calculator (2)…

Sample Question: The hypotenuse of a right triangle is 12 units. One leg is 9 units. Find the other leg.

- Open the program entitled "PYTHAG"

- Press **1** to find a LEG
- Enter **9** for OTHER LEG and **12** for HYPOTENUSE
- Solution:



- **NOTE:** The calculator does not have the functionality to reduce radicals so it will always display decimal answers if needed.

Tutorial #13

Solving Quadratic Equations

What is a Quadratic Equation?

- A **quadratic equation** is in the form $ax^2 + bx + c = 0$

  While b and/or c can be zero, a cannot! If $a = 0$, then there is no x^2 term.
  This would make the equation NOT quadratic!

- Examples are $3x^2 - 5x - 7 = 0$ or $12x^2 + 4 = 0$

- The goal is always to solve the equation for x

- These x values are called solutions, roots or x-intercepts

How Do We Solve These Equations?

There are 3 common ways to solve quadratic equations:

1) Using the **Quadratic Formula**

2) By **Factoring**

3) By **Completing the Square**

Each quadratic can have 0, 1 or 2 real solutions *OR* 2 imaginary solutions… The ACT usually focuses on real solutions, but we will introduce the concept of imaginary roots also.

How Do We Solve Quadratic Equations?

**** Many equations can be solved by more than one method ****

- This tutorial will focus on the Quadratic Formula

- For information on Factoring, please view our “Factoring A Polynomial” Tutorial

- Completing the Square can be a useful process, but it is not usually your best bet for the ACT. For information on Completing the Square, visit: https://www.purplemath.com/modules/solvquad3.htm

The Quadratic Formula!

- For any polynomial in the form a$x^2 + bx + c = 0$, the quadratic formula gives us the solution(s) for x through this formula:

$$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

- Many have heard of the Quadratic Formula, but it often causes anxiety – it does not need to!!
- You can master the use of this formula through memorization and practice!

Using the Quadratic Formula

Sample Problem: Solve the equation $8x^2 + 2x - 15 = 0$

Solution: This is a quadratic equation with $a = 8,\ b = 2\ and\ c = -15$.

We substitute these values into the quadratic formula and simplify:

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(8)(-15)}}{2(8)}$$

$$x = \frac{-2 \pm \sqrt{4 - (-480)}}{16}$$

$$x = \frac{-2 \pm \sqrt{484}}{16}$$

$$x = \frac{-2 \pm 22}{16}$$

$$x = \frac{-2 + 22}{16}\ or\ x = \frac{-2 - 22}{16}$$

$$x = \frac{20}{16}\ or\ x = \frac{-24}{16}$$

$$\boldsymbol{x = \frac{5}{4}\ or\ x = -\frac{3}{2}}$$

An Example with A Radical That Doesn't Reduce:

Sample Problem: Solve the equation $x^2 - 7x + 4 = 0$

Solution: This is a quadratic equation with $a = 1,\ b = -7\ and\ c = 4$. We substitute these values into the quadratic formula and simplify:

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(4)}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{49 - 16}}{2}$$

$$x = \frac{7 \pm \sqrt{33}}{2}$$

$$\boldsymbol{x = \frac{7 + \sqrt{33}}{2}\ or\ x = \frac{7 - \sqrt{33}}{2}}$$

This radical does not reduce, so we leave it as is.

There is nothing more we can do to simplify our answers. We can leave them as is, or we can use our calculators to obtain a decimal approximation:

$$\boldsymbol{x \approx 6.37\ or\ x \approx .63}$$

**** If you did not get these answers, make sure you followed the correct order of operations procedures. You can also use parentheses around the numerators to ensure your calculations are done correctly! ****

An Example with No "x" Term:

Sample Problem: Solve the equation $9x^2 - 20 = 0$

Solution: This is a quadratic equation with no x term and therefore no b value. That is ok! We have $a = 9,\ \ b = 0\ and\ c = -20$.

We substitute these values into the quadratic formula and simplify:

$$x = \frac{-0 \pm \sqrt{(0)^2 - 4(9)(-20)}}{2(9)}$$

$$x = \frac{0 \pm \sqrt{0 - (-720)}}{18}$$

$$x = \frac{0 \pm \sqrt{720}}{18}$$

$$x = \frac{0 \pm \sqrt{144}\sqrt{5}}{18}$$

$$x = \frac{0 \pm 12\sqrt{5}}{18}$$

$$x = \frac{12\sqrt{5}}{18} \; or \; x = \frac{-12\sqrt{5}}{18}$$

$$\boldsymbol{x = \frac{2\sqrt{5}}{3} \; or \; x = -\frac{2\sqrt{5}}{3}}$$

These can also be carefully expressed as decimals: $\boldsymbol{x \approx 1.49 \; or \; x \approx -1.49}$

An Example with No "Constant" Term:

Sample Problem: Solve the equation $6x^2 + 28x = 0$

Solution: This is a quadratic equation with no constant (number only) term and therefore no c value. That is ok! We have $a = 6,\; b = 28 \; and \; c = 0$.

We substitute these values into the quadratic formula and simplify:

$$x = \frac{-28 \pm \sqrt{(28)^2 - 4(6)(0)}}{2(6)}$$

$$x = \frac{-28 \pm \sqrt{784 - 0}}{12}$$

$$x = \frac{-28 \pm \sqrt{784}}{12}$$

$$x = \frac{-28 \pm 28}{12}$$

$$x = \frac{-28 + 28}{12} \; or \; x = \frac{-28 - 28}{12}$$

$$x = \frac{0}{12} \; or \; x = \frac{-56}{12}$$

$$\boldsymbol{x = 0 \; or \; x = -\frac{14}{3}}$$

An Example with No Real Solutions:

Sample Problem: Solve the equation $x^2 + 2x + 5 = 0$

Solution: This is a quadratic equation with $a = 1,\; b = 2 \; and \; c = 5$. We substitute these values into the quadratic formula and simplify:

$$x = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4 - 20}}{2}$$

$$x = \frac{-2 \pm \sqrt{-16}}{2}$$

A negative value under the square root indicates imaginary solutions

At this point, you can usually say there are “No Real Solutions.”

Or, if we simplify the imaginary root, the solution becomes:

$$x = \frac{-2 \pm 4i}{2}$$

$$x = \frac{-2 + 4i}{2} \; or \; x = \frac{-2 - 4i}{2}$$

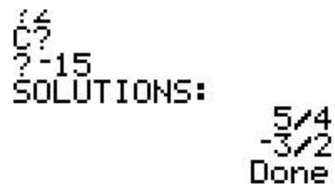
$$x = -1 + 2i \; or \; x = -1 - 2i$$

** Future tutorials will explain imaginary numbers in detail! **

Now Check It Using the Calculator:

Sample Problem: Solve the equation $8x^2 + 2x - 15 = 0$

- Open the program entitled "QUAD"
- Enter 8 for A, 2 for B, and -15 for C
- Solution:



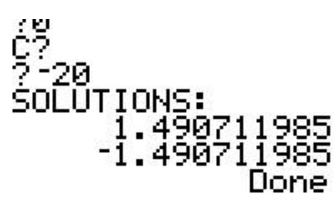
Sample Problem: Solve the equation $x^2 - 7x + 4 = 0$

- Open the program entitled "QUAD"
- Enter 1 for A, -7 for B, and 4 for C
- Solution:

- Note that the calculator doesn't display radicals, so decimal approximations are given.

Sample Problem: Solve the equation $9x^2 - 20 = 0$

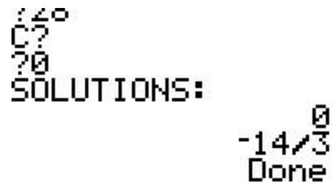
- Open the program entitled "QUAD"
- Enter 9 for A, 0 for B, and -20 for C
- Solution:



- Note that the calculator doesn't display radicals, so decimal approximations are given.

Sample Problem: Solve the equation $6x^2 + 28x = 0$

- Open the program entitled “QUAD”
- Enter 6 for A, 28 for B, and 0 for C
- Solution:



Sample Problem: Solve the equation $x^2 + 2x + 5 = 0$

- Open the program entitled “QUAD”
- Enter 1 for A, 2 for B, and 5 for C
- Solution:

- This indicates the equation has no real solutions. The calculator is unable to display imaginary answers, but can let you know no real ones exist.

Tutorial #14

Factoring a Polynomial

A Brief Overview:

FACTORING refers to the process of rewriting a polynomial expression in a different order or combination, usually to make the expression more compact

- A full explanation of factoring could take weeks (or at least hours!), so this tutorial will present the basics as well as point you to some other resources for further exploration
- We will then show you how the calculator program "QUAD" can be used to check your solutions

---

For More Information, Visit:

- https://www.mashupmath.com/blog/how-to-factor-quadratic-equations?mnc-id-sel=block-a949e3d629f78176bb57%3E-%3E1%3E-%3E1%3E-%3E1
- http://www.purplemath.com/modules/factquad.htm
- http://tutorial.math.lamar.edu/Classes/Alg/Factoring.aspx
- https://www.mathwarehouse.com/algebra/factor/how-to-factor-trinomials-step-by-step.php

FACTORING FLOWCHART

Factor out GCF

(Greatest Common Factor)

First!

2 Terms	3 Terms	4 Terms
<u>Difference of 2 Squares</u> $a^2 - b^2 = (a-b)(a+b)$ <u>Sum of 2 Cubes</u> $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ <u>Difference of 2 Cubes</u> $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$	<u>$X^2 + BX + C$</u> (x + __) (x + __) Find 2 numbers which add to **B** and multiply to **C**. <u>$AX^2 + BX + C$</u> (__x + __) (__x + __) When leading coefficient is <u>not</u> 1, use GUESS & CHECK. Find 2 numbers which multiply to **A** and 2 numbers which multiply to **C** and then use FOIL to check. **F**irst **O**uter **I**nner **L**ast	<u>Factoring By Grouping</u> If necessary, rewrite the expression so that the first 2 terms share a common factor and the last 2 terms share a common factor. Example: $8x^3 - 12x + 2x^2 - 3$ $8x^3 + 2x^2 - 12x - 3$ $2x^2(4x + 1) - 3(4x + 1)$ $(2x^2 - 3)(4x + 1)$

Now For Some Practice…

Sample Question: What is the correct factorization for the polynomial $x^2 - 11x - 60$?

Solution: First ask if there is a GCF for this problem? NO, so we proceed…

We need 2 numbers which multiply to -60 and add to -11: They are -15 and 4

The correct factorization is:

$$(\boldsymbol{x} - \mathbf{15})(\boldsymbol{x} + \mathbf{4})$$

Note that $(\boldsymbol{x} + \mathbf{4})(\boldsymbol{x} - \mathbf{15})$ is also a correct answer!

What If the Polynomial Is Not Factorable?

- There will sometimes be polynomials that cannot be factored
- An example is: $x^2 + 4x + 5$
- There is no GCF
- And, try as you might, you will not be able to come up with 2 real numbers that multiply to 5 and add to 4!
- In this case, there is nothing you can do
- Simply choose the same polynomial as the answer choice (or sometimes you may see the words *Not Factorable*)

One Other Possibility…

- Sometimes a polynomial will have a GCF but that is all….
- An example is $5x^2 - 30x - 20$
- The GCF is 5 and factoring it out gives:

$$5(x^2 - 6x - 4)$$

- At this point, you will always try to factor the remaining polynomial
- But there are no 2 real numbers that multiply to -4 and add to -6!
- $5(x^2 - 6x - 4)$ is our final answer. The polynomial can't be factored further

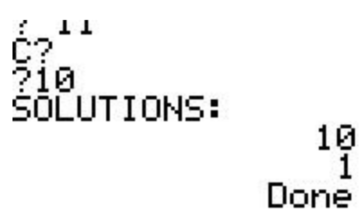
How The Calculator Can Help….

- Programs which fully factor a polynomial are usually not allowed on the ACT
- HOWEVER… if we realize the connection between Factoring and Solving a Quadratic Equation, we can get some assistance
- For example, if you are asked to solve the equation $-3x^2 + 33x - 30 = 0$, we first factor it as $-3(x - 10)(x - 1) = 0$
- Then, we would set each factor equal to zero to solve: $x = 10 \ or \ x = 1$
- This shows that if we have the solutions, we can determine the factors by taking the *opposite* of each solution
- SO…. If we have a program which solves quadratic equations, we can work backwards to check our factoring work

---

Sample Question: What is the correct factorization for the polynomial $-3x^2 + 33x - 30$?

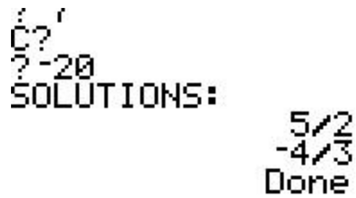
- You must first always ask yourself if there is a GCF… in this case, it is -3
- Factoring out the GCF gives the polynomial: $-3(x^2 - 11x + 10)$
- Open the program entitled "QUAD"
- Enter 1 for A, -11 for B and 10 for C
- Calculator Output:



- This confirms that the factors have to be $(x - 10)$ and $(x - 1)$
- Our final answer is $\boldsymbol{-3(x - 10)(x - 1)}$

Sample Question: What is the correct factorization for the polynomial $6x^2 - 7x - 20$?

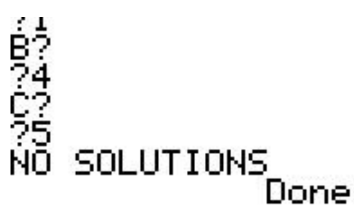
- You must first always ask yourself if there is a GCF… in this case, there is not
- Open the program entitled "QUAD"
- Enter 6 for A, -7 for B and -20 for C
- Calculator Output:



- Our solutions are $\frac{5}{2}$ and $-\frac{4}{3}$. It would seem that the factorization is $\left(x - \frac{5}{2}\right)\left(x + \frac{4}{3}\right)$. However, we do not want fractions as factors. Multiplying the first set of parentheses by 2 and the second set by 3 (the denominators of the fractions) solves the problem:
- Our final answer is $(\boldsymbol{2x - 5})(\boldsymbol{3x + 4})$

Sample Question: What is the correct factorization for the polynomial $x^2 + 4x + 5$?

- You must first always ask yourself if there is a GCF… in this case, there is not
- Open the program entitled "QUAD"
- Enter 1 for A, 4 for B and 5 for C
- Calculator Output:



- This confirms the polynomial has no real solutions and cannot be factored further. Our final answer is the original polynomial: $\boldsymbol{x^2 + 4x + 5}$

Sample Question: What is the correct factorization for the polynomial $5x^2 - 30x - 20$?

- You must first always ask yourself if there is a GCF… in this case, it is 5
- Factoring out the GCF gives the polynomial: $5(x^2 - 6x - 4)$
- Open the program entitled "QUAD"
- Enter 1 for A, -6 for B and -4 for C
- Calculator Output:

- The fact that our solutions are not integers or fractions means that it the polynomial cannot be factored further.
- Our final answer is the polynomial with the GCF factored out: $\boldsymbol{5(x^2 - 6x - 4)}$

Tutorial #15

Calculating Slope

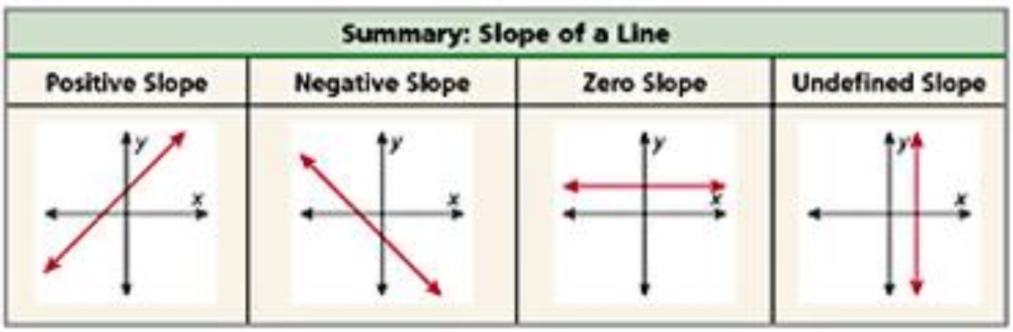
What Exactly Is Slope?

- The slope of a line is a numerical measure of how steep the line is and in what direction it rises or falls

- We calculate a slope given 2 ordered pairs on the line:

$$(x_1,\ y_1)\ and\ (x_2,\ y_2)$$

- Slopes can be positive or negative, 0 or undefined:



- Slope can be an integer, but we often see fractions

How Do We Calculate Slope?

Given 2 points $(x_1,\ y_1)\ and\ (x_2,\ y_2)$, the **Slope Formula** is:

$$\boldsymbol{Slope = m = \frac{y_2 - y_1}{x_2 - x_1}}$$

Important Slope Facts

- Parallel Lines have the same slope
- Perpendicular Lines have "opposite reciprocal" slopes

  Example: If one line has a slope of -4/5, the perpendicular line has a slope of 5/4

Common Slope Mistakes

- Inserting the variables in the wrong place (ex: x's on top)
- Adding and subtracting numbers incorrectly
- Not checking to see if you are asked to find a parallel or perpendicular slope.
- Forgetting that $\frac{0}{a\ number} = 0$ & that $\frac{a\ number}{0}$ = undefined

---

Using the Slope Formula

Sample Question: What is the slope of the line containing the points (-6, 4) and (3, -7)?

Solution: Use the slope formula with $x_1 = -6,\ y_1 = 4,\ x_2 = 3,\ \ and\ y_2 = -7$

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{-7 - 4}{3 - (-6)}$$

$$= \frac{-11}{9}$$

$$= -\frac{11}{9}$$

Sample Question: A line contains the points (0, -2) and (3, 7). What is the slope of a line perpendicular to this line?

Solution: Use the slope formula with $x_1 = 0,\ y_1 = -2,\ x_2 = 3,\ and\ y_2 = 7$

$$\text{Slope of this line} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{7 - (-2)}{3 - 0}$$

$$= \frac{9}{3}$$

$$= 3$$

Perpendicular slope = $-\ \frac{\mathbf{1}}{\mathbf{3}}$

Now Check It Using The Calculator…

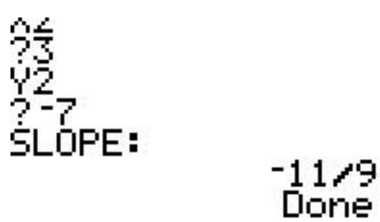
Sample Question: What is the slope of the line containing the points (-6, 4) and (3, -7)?

› Open the program entitled "SLOPE"

› Press ENTER to move past the opening screen

› Enter -6 for X1, 4 for Y1, 3 for X2 and -7 for Y2

› Solution:



Tutorial #16

Solving Systems of Equations

A Brief Overview:

SOLVING A SYSTEM OF LINEAR EQUATIONS refers to the process of finding the unique numerical solution (if one exists) for a system of 2 or more linear equations. This usually involves finding an ordered pair (x, y).

- The ACT will only ask you to solve a system of 2 equations with 2 variables!
- A comprehensive discussion of solving systems (including the multiple methods to use) is quite lengthy, so this tutorial will present the basics as well as point you to some other resources for further exploration
- The calculator program "SOLVSYST" will be the primary focus of this lesson

---

Solving Systems of 2 Equations in 2 Variables

A system of 2 equations in 2 variables looks something like this:

$$\begin{cases} 3x - 2y = 12 \\ -7x + 4y = -26 \end{cases}$$

There are 3 main ways to solve a system:

1) **GRAPHING:** This is usually time consuming and hard to do during the ACT because you don't have graph paper
2) **ELIMINATION:** This involves eliminating one of the variables by multiplying one or both equations by numerical values to cause either x or y to "cancel out"
3) **SUBSTITUTION:** This involves solving one of the equations for either x or y and substituting that expression into the other equation

*** This tutorial will focus on ELIMINATION although substitution is commonly used ***

A Quick Note…

Elimination (and the soon-to-be explained calculator program) require that BOTH equations be in standard form: $Ax + By = C$

- Standard form means the variable terms are on the **left** side of the equation (with the variables in alphabetical order) and the number is on the **right** side
- So if the system is not in standard form, rewrite it first

Example: Given the system, $\begin{cases} 5y = -2x + 16 \\ -3x - 2y = -2 \end{cases}$

- The first equation is not in standard form. So we rewrite. The correct form is:

$$\begin{cases} 2x + 5y = 16 \\ -3x - 2y = -2 \end{cases}$$

For more information…

For a general overview:

- https://www.mathwarehouse.com/algebra/linear_equation/systems-of-equation/index.php

Solving a system by GRAPHING:

- http://www.algebra-class.com/graphing-systems-of-equations.html

Solving a system using ELIMINATION:

- http://www.purplemath.com/modules/systlin5.htm

Solving a system using SUBSTITUTION:

- https://www.mathportal.org/algebra/solving-system-of-linear-equations/substitution-method.php

Now For Some Practice…

Sample Question: Use elimination to find the solution to the system of equations:

$$\begin{cases} 3x - 2y = 12 \\ -7x + 4y = -26 \end{cases}$$

Solution: We first make sure both equations are in standard form. They are! Then, decide which variable to eliminate: y seems simpler.

Multiply both sides of the first equation by 2:

$$\begin{cases} 6x - 4y = 24 \\ -7x + 4y = -26 \end{cases}$$

Now add the 2 equations together: $-x = -2$

A final algebraic step gives: $\boldsymbol{x = 2}$

So $x = 2$ for the system: $\begin{cases} 3x - 2y = 12 \\ -7x + 4y = -26 \end{cases}$

Substituting $x = 2$ into one of our original equations allows us to solve for y:

$$3x - 2y = 12$$

$$3(2) - 2y = 12$$

$$6 - 2y = 12$$

$$-2y = 6$$

$$y = -3$$

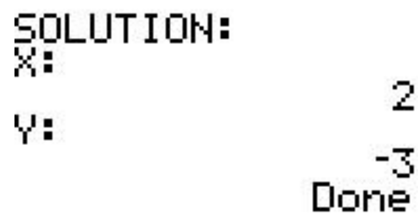
The solution to the system is $(\boldsymbol{x}, \boldsymbol{y}) = (\mathbf{2}, \ -\mathbf{3})$

Now Check It Using The Calculator….

Sample Question: Use elimination to find the solution to the system of equations:

$$\begin{cases} 3x - 2y = 12 \\ -7x + 4y = -26 \end{cases}$$

- Open the program entitled "SOLVSYST"
- Enter 3 for A, -2 for B, 12 for C, -7 for D, 4 for E and -26 for F



Tutorial #17

Evaluating Variable Expressions Using Order of Operations

What is Order of Operations?

The order of operations rules apply to evaluating arithmetic expressions. The accepted rules use the following order of application:

- Parentheses
- Exponents
- Multiplication
- Division
- Addition
- Subtraction

How Do I Remember This Order?

We frequently use the acronym "PEMDAS" to help students remember the correct order of operations. Another trick that helps is to memorize a sentence like:

"Please Excuse My Dear Aunt Sally"

Whatever method you use, what is most important is that you use the operations correctly, in the right order!

An Example:

Evaluate the following mathematical expression: $-3^2 - 2(5-8) * 9/3$

- We start by simplifying the Parentheses: $-3^2 - 2(-3) * 9/3$
- Next, comes the Exponent: $-9 - 2(-3) * 9/3$
- Next comes Multiplication and Division. We get rid of the remaining ( ) by multiplying:

$$= -9 + 6 * 9/3$$

- We do the rest of the multiplication and division steps in the order they appear:

$$= \ -9 + 54/3$$

$$= \ -9 + 18$$

- Finally we use Addition and/or Subtraction to get our final result:

$$= \mathbf{9}$$

What If There Is A Variable?

Evaluate the following mathematical expression for $x = -3$:

$$-x^3 - 2x^2 + 5|x|$$

- Our first step is to substitute -3 in for every x in the expression:

$$-(-3)^3 - 2(-3)^2 + 5|-3|$$

- In this case, the absolute value symbol is considered a "parentheses" so we start there:

$$-(-3)^3 - 2(-3)^2 + 5(3)$$

- Continue to use PEMDAS to simplify the expression:

$$= \ -(-27) - 2(9) + 5(3)$$

$$= \ 27 - 18 + 15$$

$$= \ 9 + 15$$

$$= \mathbf{24}$$

What If There Are Multiple Variables?

Evaluate the following mathematical expression for $a = -1\ and\ b = 3$:

$$ab^2 - 2b^a + 3a$$

- Our first step is to substitute -1 in for every a and 3 in for every b in the equation:

$$(-1)(3)^2 - 2(3)^{-1} + 3(-1)$$

- Use PEMDAS to simplify the expression - There is no work to do within parentheses, so start with E:

$$= (-1)(9) - 2(\frac{1}{3}) + 3(-1)$$

$$= -9 - \frac{2}{3} - 3$$

$$= -\frac{27}{3} - \frac{2}{3} - \frac{9}{3}$$

$$= -\frac{29}{3} - \frac{9}{3}$$

$$= -\mathbf{\frac{38}{3}}$$

What Could Possibly Go Wrong?!?

Unfortunately, there are many errors that students make in order of operations problems. These include not applying the order properly, making arithmetic errors, confusing negative signs, etc. So what can we do?

✓ Use the features of our approved calculator to avoid mistakes!!!

How The Calculator Can Help…

Sample Question 1: Evaluate the following mathematical expression for $x = -3$:

$$-x^3 - 2x^2 + 5|x|$$

- There is no program for this type of problem, but the calculator is still invaluable
- View a step-by-step video here: https://www.calculatesuccess.org/workbook-videos
- Solution:

```
-3→X
                    -3
-X³-2X²+5|X|
                    24
```

Sample Question 2: Evaluate the following mathematical expression for $a = -1\ and\ b = 3$:

$$ab^2 - 2b^a + 3a$$

- View a step-by-step video here: https://www.calculatesuccess.org/workbook-videos
- Solution:

```
-1→A
                    -1
3→B
                     3
```

```
AB²-2B^A+3A
      -12.66666667
Ans▸Frac
                -38/3
```

One Final Example (Using Division)

Sample Question 3: Evaluate the following mathematical expression for $r = 3 \; and \; s = -6$:

$$\frac{6rs^{-1}}{3r^2s}$$

- View a step-by-step video here: https://www.calculatesuccess.org/workbook-videos
- Solution:

```
3→R
                  3
-6→S
                 -6
```

```
6RS⁻¹/(3R²S)
     .0185185185
Ans▸Frac
               1/54
```

POST-TEST

How To Take This Post-Test

- The purpose of this post-test is to assess your skills and knowledge after:

  - Acquiring our Calculator Programs *and*

  - Completing our Tutorials Workbook

- Allow yourself about 45 minutes to take the post-test
- You are encouraged to use a calculator for all of the problems
- Take a deep breath – you're ready for this!

Before You Begin…

- Each question has exactly one correct answer.
- You have one attempt at each question.
- If you are unsure of the correct answer, take your best guess.
- You are not penalized for guessing on the ACT.

1. What is the measure of one exterior angle of a regular hexagon?

   A. 48°

   B. 52°

   C. 60°

   D. 72°

   E. Cannot be determined

2. Factor the polynomial: $3x^2 - 2x + 5$

   A. $(3x - 1)(x + 5)$

   B. $3(x - 1)(x + 5)$

   C. $(3x + 1)(x - 5)$

   D. $(3x + 5)(x - 1)$

   E. The polynomial cannot be factored

3. What is the equation of the line through the points (1, -6) and (-4, 2) in slope-intercept form?

   A. $y = -\frac{22}{5}x - \frac{8}{5}$

   B. $y = -\frac{8}{5}x - \frac{22}{5}$

   C. $y = -\frac{2}{3}x - \frac{22}{3}$

   D. $y = \frac{5}{8}x + 22$

   E. $y = 8x - \frac{22}{5}$

4. One leg of a right triangle measures 10 cm. The hypotenuse is 16 cm. What is the length of the other leg, rounded to the nearest hundredth?

   A. 12.49 cm

   B. 12.52 cm

   C. 15.35 cm

   D. 18.64 cm

   E. 18.87 cm

5. Completely factor the expression: $4x^2 - 16x - 20$

A. $-4(-x+5)(x-1)$

B. $-4(x+5)(x-1)$

C. $4(x-5)(x+1)$

D. $4(x-1)(x+5)$

E. $(4x-20)(x+1)$

6. What is the greatest common factor for the expressions: $81a^3b^4 \ and \ 36ab^5$

A. $3ab$

B. $3ab^4$

C. $3a^3b^4$

D. $9ab^4$

E. $9a^3b^5$

7. What number is 24% of 150?

A. 3.6

B. 6.25

C. 36

D. 36.24

E. 62.5

8. What is the sum of the measures of the interior angles for a 15-sided polygon?

A. 360°

B. 1080°

C. 1440°

D. 1980°

E. 2340°

9. Find the product of the solutions to the quadratic equation: $8x^2 - 18x - 35 = 0$

A. $-\frac{35}{8}$

B. $-\frac{9}{4}$

C. $\frac{9}{4}$

D. $\frac{35}{8}$

E. There are no real solutions to the equation

10. W is the midpoint of segment YZ. If the coordinates of point Y are (0, -5) and the coordinates of point Z are (-6, 3), what are the coordinates of point W?

A. (-12, -11)

B. (-3, -1)

C. (-3, 4)

D. (3, -1)

E. (6, -13)

11. An antiques dealer buys a rare coin for $120. He wants to make a 20% profit when he sells it. How much should he advertise the coin for?

A. $24

B. $96

C. $144

D. $156

E. $288

12. What is the y-coordinate in the solution of the system: $\begin{cases} 5x - 2y = 14 \\ 8x + 4y = 26 \end{cases}$

A. $-\frac{7}{2}$

B. $-\frac{1}{2}$

C. $\frac{1}{2}$

D. 3

E. $\frac{7}{2}$

13. Evaluate the expression: $-6|-2| + 4^{-2} * 3 - \frac{1}{2}$

A. -28

B. $-\frac{197}{16}$

C. $-\frac{191}{16}$

D. $-\frac{73}{2}$

E. $\frac{187}{16}$

14. What is the slope of the line -5x + 15y = 30 ?

A. -2

B. $-\frac{1}{2}$

C. $-\frac{1}{3}$

D. $\frac{1}{3}$

E. 2

15. Find the solutions to the quadratic equation: $x^2 - 9x - 8 = 0$

A. No real solutions

B. -9.815 and 0.815

C. -3.245 and 1.625

D. -1.625 and 3.245

E. -0.815 and 9.815

16. M is the midpoint of segment ST. If the coordinates of point S = (5, -2) and the coordinates of point M = (-1, 5), find the coordinates of point T.

A. $(-11, -9)$

B. $(-7, 12)$

C. $\left(2, \frac{3}{2}\right)$

D. $\left(3, \frac{7}{2}\right)$

E. $(12, -7)$

17. A regular polygon has interior angles which each measure 156 degrees. How many sides does the polygon have?

A. 12

B. 13

C. 14

D. 15

E. 16

18. Jordan has a 92% average in his history class after 5 exams. What score would he need on the sixth exam to guarantee himself a 90% average?

A. 80%

B. 85%

C. 89%

D. 90%

E. 94%

19. Find the distance between the points (-7, 4) and (9, 8) to the nearest tenth.

A. 4.9

B. 7.8

C. 9.1

D. 12.2

E. 16.5

20. A line contains the points (4, -11) and (0, 5). What is the slope of a line parallel to this line?

A. -4

B. $-\frac{1}{4}$

C. 0

D. $\frac{1}{4}$

E. 4

21. What is the measure of one interior angle of a pentagon?

A. 60°

B. 72°

C. 108°

D. 120°

E. Cannot be determined

22. What are the solutions to the quadratic equation: $5x^2 - 30 = 0$?

A. No real solutions

B. -6.449 and 6.449

C. -6 and 0

D. -2.449 and 2.449

E. 0 and 6

23. Evaluate the expression when p = 3 and q = -1: $\frac{pq^3}{-9qp^2}$

A. -3

B. $-\frac{1}{9}$

C. $-\frac{1}{27}$

D. $\frac{1}{3}$

E. 27

24. Find the distance between the points (-3, 5) and (12, -3).

A. $\sqrt{59}$

B. 8

C. $\sqrt{85}$

D. 14

E. 17

25. The midpoint of a segment is (3, -1/2). If one endpoint is (-2, 5), find the coordinates of the other endpoint.

A. $(-7,\ -6)$

B. $(\frac{1}{2},\ \frac{9}{4})$

C. $\left(\frac{3}{2},\ 6\right)$

D. $(8,\ -6)$

E. $(8,\ 6)$

26. Find the y-intercept for the line containing the points (-7, 2) and (4, -1).

A. -11

B. $-\frac{1}{11}$

C. 0

D. $\frac{1}{11}$

E. $\frac{3}{7}$

27. What is the greatest common factor of 125 and 320?

A. 3

B. 5

C. 10

D. 15

E. 25

28. Find all real solutions to the equation: $9x^2 + 27x = 0$

A. -3

B. -3 and 0

C. -3 and 3

D. 0 and 3

E. No real solutions

29. 45 is what percent of 360?

A. 2.5%

B. 12.5%

C. 15%

D. 162%

E. 800%

30. A candy company has sales totaling $42,350 after 11 months. How much would they need to earn in their 12th month to make the mean monthly sales $4100?

A. $3850

B. $4100

C. $4500

D. $5245

E. $6850

31. Evaluate the expression when c = -2 and d = -1: $cd^{-c} - 2c^2 + 3d$

A. -19

B. -13

C. -7

D. -6

E. -2

32. Completely factor the expression: $-12x^2 + 18x - 30$

A. $-6(2x^2 - 3x + 5)$

B. $-6(2x^2 + 3x + 5)$

C. $-6(2x - 5)(x + 1)$

D. $-3(4x^2 - 6x + 10)$

E. The polynomial cannot be factored

33. What is the sum of the measures of the exterior angles of a 24-sided polygon?

A. 360°

B. 720°

C. 1800°

D. 2160°

E. 3960°

34. What is the x-intercept of the line 3x - 7y = 21 as an ordered pair?

A. (-7, 0)

B. (-3, 0)

C. (0, -3)

D. (0, 7)

E. (7, 0)

35. What is the midpoint of the segment with endpoints (-2, 7) and (6, -4)?

A. $\left(-\frac{11}{2},\ 4\right)$

B. $\left(-2,\ \frac{3}{2}\right)$

C. $\left(2,\ -\frac{3}{2}\right)$

D. $\left(2,\frac{3}{2}\right)$

E. $(4,-\frac{11}{2})$

36. What is the correct factorization for the polynomial: $x^2 + 6x - 72$

A. $(x - 12)(x + 6)$

B. $(x - 9)(x - 8)$

C. $(x - 9)(x + 8)$

D. $(x - 7)(x + 13)$

E. $(x - 6)(x + 12)$

37. Zack wants to buy a car with a sticker price of $24,000. The dealership offers him a 12% discount. What is the discounted price?

A. $2,880

B. $14,400

C. $21,120

D. $23,520

E. $26,880

38. What is the slope of the line containing the points (-2, -6) and (3, 5)?

A. $-\frac{11}{5}$

B. $-\frac{5}{11}$

C. $\frac{1}{5}$

D. $\frac{5}{11}$

E. $\frac{11}{5}$

39. Solve the quadratic equation: $3x^2 - 5x + 11 = 0$

A. No real solutions

B. -6.653 and 1.653

C. -2.922 and 1.255

D. -1.653 and 6.653

E. -1.255 and 2.922

40. What is the solution to the system of equations: $\begin{cases} -4x + 7y = -15 \\ 3x - 5y = 11 \end{cases}$

A. (-2, 1)

B. (-1, 2)

C. (0, -3)

D. (2, -1)

E. (3, -1)

41. A right triangle has legs of length 24 and 45. What is the length of the hypotenuse?

A. 48

B. 50

C. 51

D. 58

E. 62

42. What is the greatest common factor of 144, 162 and 207?

A. 1

B. 3

C. 6

D. 9

E. 18

43. On Monday, Karen did 45 push-ups. On Tuesday, she did 39 push-ups. On Wednesday, she did 27 push-ups. On Thursday, she did 50 push-ups. How many push-ups should she do on Friday to make her daily average 35?

A. 14

B. 27

C. 35

D. 45

E. 48

44. The sum of the interior angles of a polygon is 3060 degrees. How many sides does the polygon have?

A. 9

B. 15

C. 19

D. 20

E. 24

45. 25 is 12% of what number?

A. 3

B. $\frac{400}{9}$

C. $\frac{625}{3}$

D. $\frac{700}{3}$

E. 300

46. A regular polygon has exterior angles which each measure 18 degrees. How many sides does the polygon have?

A. 18

B. 20

C. 22

D. 24

E. 26

47. Evaluate the expression for x = -2: $-4x^2 + 5x^3 - 2x + 3$

A. -57

B. -49

C. -17

D. 23

E. 33

POST-TEST SOLUTIONS

1.	C	13.	B	25.	D	37.	C
2.	E	14.	D	26.	D	38.	E
3.	B	15.	E	27.	B	39.	A
4.	A	16.	B	28.	B	40.	D
5.	C	17.	D	29.	B	41.	C
6.	D	18.	A	30.	E	42.	D
7.	C	19.	E	31.	B	43.	A
8.	E	20.	A	32.	A	44.	C
9.	A	21.	E	33.	A	45.	C
10.	B	22.	D	34.	E	46.	B
11.	C	23.	C	35.	D	47.	B
12.	C	24.	E	36.	E		

I've Taken the Post-Test - Now What?!

- We hope your score improved from the Pre-Test!
- We also hope it took you less time to complete it
- We stand behind our programs and lessons and believe they help explain topics and provide more time to achieve greater success on the ACT Math Section
- If you did not see your desired improvement:
    - Work through our tutorials again – Sometimes it takes a few tries for the material to sink in
    - Contact us. We want to help you succeed!

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